THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME VI AUGUST 1897 NUMBER 2

ON THE LAW OF SPECTRAL SERIES.

By T. N. Thiele.

Having long been occupied with investigations on the law of spectral series, I am in a position to affirm that if this problem seems beyond the reach of mathematical *deduction*, it is certainly very difficult to obtain by *induction* a sufficiently approximate general solution by means of the measures of spectra which we possess at the present time. The first steps in this direction were relatively easy, but in order to develop ultimately what may be considered known at the present time regarding the simplest series and double or triple lines and series, it is necessary to study with great care the not infrequent small anomalies which prevent us from considering these ideas as other than very imperfect approximations. It is necessary also to investigate the very complex cases in which the deviations present themselves in great detail, and from which we may be able to obtain, though at the expense of great labor, new and important information. For this problem is a very troublesome one, and those who occupy themselves with it cannot hope to make, so far as my experience goes, those little discoveries which relieve tedious investigations. In fact, one's fundamental assumptions often give way before the constant criticism to which they are exposed.

The more or less complete resolution of spectra into series

may, however, be accomplished before the general law of series is discovered, and I desire to contribute in some small degree to this useful work. For the present I confine myself to a few general remarks, all of them, with perhaps a single exception, of a negative character.

The single established fact in the present theory of series, the only one which my investigations have more and more tended to confirm, is that the law which expresses the wave-length λ of the lines of a series as a function of the series-number n of the lines must have the form

$$\lambda = f\left[(n + c)^2\right], \qquad (1)$$

where c is a constant, which I shall call the phase of the series. Excepting certain developments in series which are evidently but slightly convergent, all the formulæ hitherto proposed are special cases of this general form. Taking this law as a fundamental hypothesis, I accept all its consequences, some of which seem to have been rather neglected up to the present time.

Giving n successively all real integral values, it is evident that λ must have at least one maximum and one minimum value, representing what are called the heads of the series. According to formula (1) there is an important difference between these values, which in reality exists, and which must be indicated by a suitable nomenclature. In the neighborhood of $\lambda_0 = f(0)$ a finite number of lines are united into an ordinary *head*, but near $\lambda_\infty = f(\infty)$ an infinite number of lines are generally crowded into a finite space. This last form clearly occurs in line spectra, and I propose to call λ_∞ the *tail* of the series.

The difference between band and line spectra may be expressed as follows: In band series we ordinarily observe the heads, while these are invisible in line series, either on account of their extreme position or because of the poverty of lines in these spectra. As regards the tails, I have already remarked that they are found in line spectra, but they have not been noticed in band spectra. One is tempted to consider this invisibility as a characteristic feature of band spectra; but I have

suspected the presence of certain discontinuities in one of M. Rydberg's photographs of the spectrum of cyanogen: certain sudden interruptions of the fogged gray background which might be regarded as the tails of the series in this spectrum, though it is impossible to refer each one to its series. This is unfortunate, for the presence of both the head and tail of the same series would probably greatly facilitate the discovery of the law of the series.

The most important consequence of our hypothesis, $\lambda = f[(n+c)^2]$, is that it is necessary to take into account not only the lines corresponding to positive values of n, but also those obtained when $n < 0$. In other words a series must in general be composed of two groups of lines, each of which would ordinarily be called a series. I prefer to put it that the positive branch of each series must be accompanied by a negative branch of the same series, having the same head and the same tail and being represented alternately by a line in each interval of the other branch. These two branches may exactly coincide, in which case the phase of the series must be either $c = 0$ or $c = \frac{1}{2}$ (evidently c must be defined so as to include only fractions properly so called).

That each series consists of two branches is not merely a logical consequence of the equation $\lambda = f[(n+c)^2]$: it is a well-established fact in many spectra, notably in the case of band spectra. Here the phenomenon is so regular that in cases where such double branches are not observed (*e. g.*, in the spectrum of nitrogen) it may be concluded that $c = 0$ or $c = \frac{1}{2}$ with great probability that this supposition will be confirmed by subsequent calculations.

However, it is not safe to assume that both branches of each series will invariably be seen. In certain instances, illustrated in the spectrum of carbon, the two branches are equally intense, with the maximum of intensity in the head. But in other spectra the intensity is distributed with much less equality and regularity. For example, the great series discovered by Professors Kayser and Runge in the third band of cyanogen has

another branch which is still more intense near the common head $\lambda_0 = 3883.55$; the intensity of this latter branch, however, falls off more rapidly than does that of Kayser and Runge's branch, so that it has totally disappeared at $n = 100$, while the first branch remains visible when n is nearly twice as great. Frequently the two branches are confused near the head, and when this duplicity has been resolved, the difference between the positive and negative series-numbers of the lines of the two branches may be great enough to account for the extreme faintness or invisibility of one of them. Although standing side by side, they belong to entirely different parts of the series.

Sometimes there are numerous branches. Instead of two, three or four or even more are found, none of which show coincidences of their ordinary lines. In such an embarrassment of riches it becomes necessary, in order to separate them from the others, to discover pairs of branches whose not very regular spacing seems to indicate that if sufficiently prolonged they would present coincidences other than those of the head or tail. It then becomes a question, not of the branches of a single series, but rather of two neighboring and parallel series, analogous to doublets in line spectra, although they do not obey the well-known criterion of the latter: $\frac{1}{\lambda_n} - \frac{1}{\lambda'_n} =$ const.

Line spectra also offer many examples of series having two branches. We may probably cite all the double series except the doublets properly so called, for example the double series of the alkalis. In the case of hydrogen the series discovered by Professor Pickering seems to be the negative branch of that which has been regarded as the sole and complete series of this element. Then for hydrogen the phase must be $c = \frac{1}{4}$, for in this way one of the branches comes by interpolation in the middle of the other's intervals, without that irregular progression generally presented by series, except the double series $c = 0$ and $c = \frac{1}{2}$, and particularly the series $c = \pm \frac{1}{4}$.

In line spectra we have even less reason than in the case of band spectra to expect the regular appearance of the two

branches. That they perhaps differ very decidedly in intensity or even in appearance is not at all surprising, for in line spectra they in reality belong to entirely different parts of the series. On the side of the tail they are separated by an infinitude of invisible lines; on the side of the head by a finite and even small number of lines, which, nevertheless, make a double passage across the invisible regions of the spectrum. Moreover, even in these spectra an analogous phenomenon has been remarked: the series of diffuse lines and those of sharp lines.

The question now arises whether the double series ordinarily found in metallic spectra may not also be regarded as constituting a single series. It is known that the tails of the members of the pairs coincide, but nothing is known regarding the heads; it may be supposed, however, that they also coincide, as the ordinary lines of these series are never coincident. In each interval of the series there is always found a single line belonging to the other; these apparently always occupy the same position in the interval, as must be the case if both series can be expressed by the same formula $\lambda = f\left[(n + c)^2\right]$, positive values of n referring to one series and negative values to the other.

While I do not wish to assert that these pairs of series are in reality merely branches of single series, I desire to point out that at the present time this is the most probable supposition, which should not be abandoned except in the face of the strongest arguments. For reasons already indicated it is clear that such arguments are not to be found in differences in the intensity and the appearance of the lines.

But if by chance there were to be discovered in the spectra of certain metals four corresponding branches, two of them sharp and the remaining two diffuse, the argument against the general connection of these pairs as branches of single series would be strengthened, especially if at that time our knowledge of the general law of series had so far advanced as to permit us to determine the phase of each branch independently. For until the nearly definitive discovery of the true law of series has been

made, I suppose the phase could be well determined only by indirect interpolation of the positions of the lines of one of the branches in the intervals of the other. Thus only a systematic variation in this progression of values of the phase can demonstrate in a satisfactory manner the non-correspondence of the two branches of the series.

In this JOURNAL (**4**, 369) Professor Pickering has used a formula for the series in which four arbitrary constants can be introduced. Its slightly modified algebraic form

$$\lambda = \frac{\lambda_0 a + \lambda_\infty (n+c)^2}{a + (n+c)^2} \tag{2}$$

requires that $\lambda = \lambda_0$ when $n + c = 0$ and $\lambda = \lambda_\infty$ when $n = \infty$.

This formula is a generalization of Rydberg's formula, just as the latter is a generalization of Balmer's. It is evidently a special case of $\lambda = f\left[(n+c)^2\right]$. We may also write

$$\frac{\lambda - \lambda_0}{\lambda_\infty - \lambda} = \frac{(n+c)^2}{a} \qquad \text{or} \qquad \frac{\lambda^{-1} - \lambda_0^{-1}}{\lambda_\infty^{-1} - \lambda^{-1}} = \frac{\lambda_\infty}{\lambda_0}\,\frac{(n+c)^2}{a};$$

the algebraic form and three of the constants, viz., the phase, the head, and the tail, remain the same if in place of wave-lengths, λ, we prefer to employ wave-frequencies, λ^{-1}.

In these rather delicate investigations this formula is very useful. It possesses the great advantage of having easily determinable constants. If for four series-numbers m, n, p, and q, one of which is arbitrary, the wave-lengths M, N, P, and Q are known, we have, after eliminating three constants, the final equation

$$\frac{M-N}{M-P}\cdot\frac{P-Q}{N-Q} = \frac{(m+c)^2-(n+c)^2}{(m+c)^2-(p+c)^2}\cdot\frac{(p+c)^2-(q+c)^2}{(n+c)^2-(q+c)^2}$$

whence

$$(m+n+p+q+4c)^2 = (m-n+p-q)^2 + \frac{4(m-q)(n-p)}{1 - \dfrac{M-N}{M-P}\cdot\dfrac{P-Q}{N-Q}\cdot\dfrac{m-p}{m-n}\cdot\dfrac{n-q}{p-q}}.$$

Having found c by this formula or in some other way, we must first of all search for the value of λ_∞ corresponding to the

tail of the series, using three wave-lengths and the corresponding series-numbers. Thus we have

$$\frac{(p+c)^2-(m+c)^2}{N-\lambda_\infty}=\frac{(p+c)^2-(n+c)^2}{P-N}-\frac{(n+c)^2\,(m-+c)^2}{N-M}.$$

Analogous formulæ for λ_0 and a are not required; it is better to use at once the value of λ_∞ found in the transformation of the wave-lengths λ into $(\lambda-\lambda_\infty)^{-1}$, for

$$(\lambda-\lambda_\infty)^{-1}=\frac{a+(n+c)^2}{a\,(\lambda_0-\lambda_\infty)} \qquad (3)$$

is a complete function of the second degree, from which the constants a and λ_0 (and c as a check), as well as the table of wave-lengths, are easily obtained by interpolation.

Unfortunately, as I in fact suspected a year ago, this formula is not the true law of series. When tested with many observed series which are long and rich in lines, it shows decided deviations of a fairly constant and systematic character. It invariably places the tail at too great a distance from the head and from the observed λ. The other constants are also unsatisfactorily determined by Professor Pickering's formula: in some cases of band series I have found errors in the phase amounting to several units. Nevertheless the usefulness of this formula is not confined to the recognition of lines in a new series. Its degree of approximation is nearly always sufficient to serve as a point of departure, and frequently the series are simple or short enough to be represented without sensible error by this remarkable formula.

For more precise investigations Professor Pickering's formula may be replaced by an algebraic series of which it represents only the first term. But the succeeding terms must be chosen in such a manner as to sacrifice none of the value of Professor Pickering's formula. If, for example, we were to substitute for $(\lambda-\lambda_\infty)^{-1}$ a complete function of $(n+c)^2$,

$$(\lambda-\lambda_\infty)^{-1}=a_0+a_1(n+c)^2+a_2(n+c)^4+a_3(n+c)^6+\ \ldots\ .$$

such a simple and convenient formula would in general be too slowly convergent. For the long series of band spectra it might

be very useful as far as the region of the tail, but for line spectra it is necessary that the function (3) shall remain of the second degree for large values of $(n+c)$.

A better form would be

$$\left.\begin{aligned}\lambda &= \lambda_\infty + \frac{a_1}{(n+c)^2+C_1} + \frac{a_2}{(n+c)^2+C_2} + \dots \\ &= \frac{p_0 + p_1(n+c)^2 + \dots + p_r(n+c)^{2r}}{q_0 + q_1(n+c)^2 + \dots + q_r(n+c)^{2r}}\end{aligned}\right\} \qquad (4)$$

This formula, which like that of Professor Pickering has the same general form for λ and for λ^{-1}, is the only one that I can recommend. It is not very convenient, the principal difficulty being the determination of the phase c, which can be obtained only by hypothesis or by indirect interpolations. Subsequently λ, $\lambda(n+c)^2$, ... and $\lambda(n+c)^{2r}$ must be developed in complete functions of the common argument $(n+c)^2$ (*e. g.*, by Newton's general interpolation formula), and values of q_0, q_1, ... q_r must be found such that the sum

$$q_0\lambda + q_1\lambda(n+c)^2 + \dots + q_r\lambda(n+c)^{2r}$$

shall not be of higher degree than the second. It further remains to develop this function, *i. e.*, $p_0 + p_1(n+c)^2 + \dots + p_r(n+c)^{2r}$ and to compute a table of its values, or to solve the equation

$$0 = q_0 + q_1(n+c)^2 + \dots + q_r(n+c)^{2r},$$

the roots of which are the constants $-C_1$, ... $-C_r$ of the first form of (4), and finally to compute its other constants, a_1, ... a_r.

In these computations it is necessary to employ good observations and to subject them to rigorous criticism by means of preliminary approximate computations. For small errors may frequently lead to a positive root such that a denominator $(n+c)^2+C_1$ may pass through zero between two observed lines, in which case the whole computation would be compromised.

As an illustration of these operations, I return to the question of the relation between sharp and diffuse series as branches

of a single series. I have chosen as an example the admirable observations of Professors Runge and Paschen on the spectrum of helium,[1] and in particular the two series or branches of which the wave-lengths reduced to a vacuum are given in the second column of the following table.

n	λ	$L'=\sqrt{\frac{1}{\lambda-\lambda_\infty}-\frac{1}{\lambda_0-\lambda_\infty}}$	$\Delta L'$	$L'-L$	$c=\frac{L'-L}{2\Delta L'}$
3	7067.61	.0238188		.0026253	.14884
3'	5877.477	.0264441			
4	4714.668	.0326440	.0088209	26210	.14856
4'	4472.878	.0352650			
5	4122.115	.0414774	88222	26098	.14790
5'	4027.457	.0440872			
6	3868.689	.0503064	88228	26036	.14754
6'	3820.813	.0529100			
7	3734.045	.0591329	88224	25995	.14732
7'	3706.185	.0617324			
8	3653.140	.0679590	88216	25952	.14710
8'	3635.408	.0705542			
9	3600.480	.0767782	88220	25980	.14724
9'	3588.430	.0793762			
10	3564.123	.085605	8815	2586	.1467
10'	3555.590	.088191			
11	3537.954	.094425	8830	2596	.1470
11'	3531.636	.097021			
12	3518.47	.103238	8835	2618	.1482
12'	3513.63	.105856			
13	3503.45	.112115	8825	2565	.1453
13'	3499.76	.11468			
14	3491.75	.12095	8815	2545	.1444
14'	3488.85	.123495			
15	3482.58	.12961	8825	2710	.1535
15'	3480.08	.13232			
			887		
16'	3472.91	.14119			
			884		
17'	3467.01	.15003			
			822		
18'	3462.4	.15825			
			.00955		
19'	3457.9	.1678			

In the first column are given the series-numbers, 3′ to 19′ referring to the sharp series and 3 to 15 to the diffuse series. These series-numbers are identical with those of Professors Runge and Paschen and have been confirmed by preliminary

[1] This JOURNAL, 3, 4.

computations made with the aid of Professor Pickering's formula. I have reason to believe that they really represent the whole numbers which most closely correspond to the true values of $n + c$. With the values $\lambda_\infty = 3422$ and $\frac{1}{\lambda_0 - \lambda_\infty} = -.000292036$ obtained from a preliminary computation, I have calculated the third column $L' = \sqrt{\frac{1}{\lambda - \lambda_\infty} - \frac{1}{\lambda_0 - \lambda_\infty}}$, in such a manner that it would be a linear function $= \alpha + \beta n$, if Professor Pickering's formula were exact for the sharp series. The fourth column, $\Delta L'$, containing the first differences of the third, shows how well founded this hypothesis is. The fifth column contains the differences of the third column for the same series-numbers 3 and 3′ . . . . 15 and 15′ of the two series (sharp and diffuse). Divided by twice the numbers corresponding to the fourth column $2\Delta L'$, these give us the values of the phase c which are found in the sixth column.

There is evidently an *approximate* constancy in these values of the phase; a conclusion based on the varying value of c that the two series are unrelated would not be justifiable. The slight variation indicated at the head of the sixth column is perhaps only a consequence of the approximate nature of our computation. To enable us to judge of this the degree of approximation must be carried to such a point that the two series, considered as branches of a single series, may be sufficiently well represented by the same formula, for example of the form (4).

For the value of the phase required in this computation, I started with $c = 0.147$, in the selection of which I was guided by the sixth column of the table. With this there resulted too great a difference between the computed and observed values corresponding to $n = 3$ and $n = 4$. For $n = 8$ . . . . $n = 15$ the agreement was satisfactory.

In a second computation, using the value $c = 0.150$, I found

$$\lambda = 3421.676 + \frac{13719.472}{(n+c)^2 - 4.385604} - \frac{632.574}{(n+c)^2 + 18.79675}.$$

The particular values of this function, with the differences between the observed and computed values, are given in the following table:

$n+c$	λ comp.	$o-c$	$n+c$	λ comp.	$o-c$
−14.85	3482.509	+ .07	3.15	5877.477	.000
−13.85	3491.868	− .12	4.15	4472.868	+.010
−12.85	3503.590	− .14	5.15	4027.457	−.017
−11.85	3518.554	− .08	6.15	3820.813	.000
−10.85	3538.093	− .139	7.15	3706.176	+.009
− 9.85	3564.313	− .190	8.15	3635.404	+.004
− 8.85	3600.719	− .239	9.15	3588.432	−.002
− 7.85	3653.506	− .366	10.15	3555.574	+.016
− 6.85	3734.581	− .536	11.15	3531.636	−.009
− 5.85	3869.561	− .872	12.15	3513.656	−.03
− 4.85	4123.641	−1.526	13.15	3499.780	−.02
− 3.85	4717.376	−2.708	14.15	3488.844	+.01
− 2.85	7069.532	−1.92	15.15	3480.067	+.01
− 1.85	−10851.859		16.15	3472.914	.00
− 0.85	− 356.046		17.15	3467.006	.00
+ 0.15	+ 243.634		18.15	3462.068	+.3
+ 1.15	− 1088.710		19.15	3457.899	.0
+ 2.15	+61308.022				

The branch of the positive values of $n+c$ is thus very well represented by this formula, while the other branch only shows differences which are sufficiently small to permit us, by the use of $\frac{d\lambda}{dn} = \frac{d\lambda}{dc}$, to draw conclusions regarding the possibility of obtaining a better agreement by the use of some other hypothesis concerning c. The reply is in the negative: it is impossible to appreciably diminish these differences, taken as a whole, without subsequent complication of the formula. While the addition of a term $\frac{a}{(n+c)^2 + C_3}$ would certainly cause the difference for $n + c = -2.85$ to disappear, and at the same time decrease the other differences, it may be seen that there would nevertheless remain a marked systematic deviation.

In spite of the remarkable correspondence of these two series, I must therefore deny their unity.

Both may be represented as having exactly coincident branches, *i. e.*, by the aid of formula (4) in which $c = 0$ or $c = \frac{1}{2}$.

Or again, if we suppose that the two branches corresponding to negative values of n are too faint to be observed, they may be represented by Professor Pickering's formula, placing:

Sharp series	Diffuse series
$\lambda_\infty = 3421.967$	$\lambda_\infty = 3421.967$
$\lambda_0 = 0.052$	$\lambda_0 = 58.070$
$a = -3.757$	$a = -3.825$
$c = -0.001542$	$c = -0.28824$

THE OBSERVATORY, COPENHAGEN.
June 1897.

ON THE RELATIVE BEHAVIOR OF THE H AND K LINES OF THE SPECTRUM OF CALCIUM.[1]

By Sir William Huggins and Lady Huggins.

The remarkable relative behavior of the lines in the spectra of certain substances as they appear at and near the Sun's limb, and in the atmospheres of stars of different classes, has long been before our minds as a problem of great interest, which there is reason to believe is capable of solution by the methods of the laboratory, and on which we have worked from time to time for many years. Without waiting for the results of other researches which are in progress, we think that it is desirable to put on record some definite results on the behavior of the lines of calcium, which appear to us to be conclusive and of great importance in forming a correct interpretation of many solar and stellar phenomena.

As early as 1872 Professor Young from a few weeks' work at Sherman on the spectra of the chromosphere and of the prominences, was able to point out that "the selection of lines seems most capricious ; one is taken and another is left, though belonging to the same element, of equal intensity, and close beside the first." Especially he noticed that while the H and K lines of calcium are almost always observable, the strong blue line as well as the other lines of this metal are very seldom seen. In his table of the chromospheric lines Professor Young gives for the frequency of this strong blue line the small number 3 ; while for the frequency of H and K he gives respectively the high numbers 75 and 50.

From 1863, when I mapped the spectrum of calcium with a strong spark from metallic calcium (*Phil. Trans.*, 1864, p. 139) I have constantly used the lines of calcium as a comparison spectrum in stellar work. The experience was familiar to me that as the quantity of calcium salt on the electrodes became very

[1] Read before the Royal Society.

small, H and K continued strong even when the other calcium lines had almost disappeared. The suggestion then occurred to me that this behavior of the lines might furnish a clue to the phenomena which take place near the Sun's limb.

We were encouraged to use this experience as a guiding thought in the experiments about to be described, by the consideration that in the higher solar regions, where H and K appeared alone of the calcium lines, the density must be much less than at the lower level of the reversing layer. It seemed very probable that in the simple fact of difference of density lay the true explanation of the modifications of the calcium spectrum as they are presented to us in solar and stellar phenomena.

The problem before us was, therefore, to find out by experiments in the laboratory, under what conditions the lines of calcium other than the lines H and K, and in particular the strong blue line at 4226.9, were so greatly enfeebled relatively to H and K, that they became quite insignificant, and if possible, disappeared altogether from the spectrum, leaving the very simple spectrum of the two lines H and K, or nearly so.

Professor Lockyer states that: "Some of the substances which have been investigated, including iron, calcium, and magnesium, have probably a definite spectrum, consisting of a few lines, which can only be completely produced at a temperature higher than any which is at present available in laboratory experiments." (*Proc. R. S.*, **61**, 205.)

In the case of calcium:

"(4) A spectrum consisting of the two lines at 3706.18 and 3737.08 and the H and K lines, corresponding to a temperature higher than the average temperature of the spark, as before explained." (*Ibid.*, p. 161.)

Such a spectrum was not actually obtained, but experiments with a large intensity coil suggested that by a still greater increase of intensity of the spark such a simple spectrum might appear. The intensity of the strong blue line was reduced to one half of H and K. (*Ibid.*, Table, p. 162.)

Kayser and Runge found 106 lines of the calcium spectrum

to belong to the series of triplets; among the remaining lines they pointed out pairs with constant differences of wave-frequencies. Notably H and K, with a difference of wave-frequency of 222.9, and the more refrangible pair at 3737.08 and 3706.18, with a difference 223.1.

Messrs. Humphreys and Mohler in their experiments on the effect of pressure on the wave-lengths of metallic lines, found that in the case of calcium, the H and K lines were shifted only one-half as much as the blue line at 4226.9. We know far too little to justify us in forming any theoretical conclusions from this peculiarity of behavior. Indeed there are no certain reasons why the lines of any substance should be equally shifted.

It is well known that calcium, in common with nearly all substances, gives a more complex spectrum under the conditions of the arc and spark than under those of a flame. Now in the Fraunhofer lines we have, as first shown by Kirchhoff and Bunsen, absorption spectra of the elements which correspond, speaking broadly, with those of the bright-lined spectra of the same substances as they are produced by the spark. In order, therefore, to study the modifications which the calcium undergoes in the higher regions of the chromosphere, in the prominences, and possibly in lower parts of the corona, as well as in the atmospheres of stars of different orders, it was clearly desirable that we should start with an ordinary spark spectrum. It was suggested to us strongly by the known rarer state of the gases in the regions above the photosphere, as well as by my long experience with the behavior of calcium in comparison spectra, that the modifications of the calcium spectrum which we were seeking would be likely to show themselves under conditions of greatly reduced density of the calcium vapor.

EXPERIMENTS.

For reasons which will be obvious later on, we elected to use throughout the experiments a spark of very small intensity.

1. The break of a 6-inch Apps coil was fixed at the position of smallest acting force of the spring. So much battery power

only was employed as would be just sufficient to move the break. Under these conditions, when a jar was not in connection, the feeble spark would not pass when the distance between the points exceeded 1¼ inches.

2. In all the experiments a jar was intercalated.

3. The same length of exposure, a very short one of a second and a half, sufficient to bring out only the strongest lines of the spectrum, was used in each experiment.

4. Two sets of similar experiments were made; in one case with electrodes of platinum and in the other with electrodes of iron. In the latter case the chief lines of iron were present with those of calcium.

METHOD ADOPTED FOR REDUCING THE DENSITY OF THE CALCIUM VAPOR.

(*a*) The spark was taken between electrodes of metallic calcium. It was assumed, as was confirmed by the appearance of the spark, that with metallic calcium for electrodes, the largest amount of calcium vapor would be present.

(*b*) The tips of the electrodes, iron or platinum, were slightly moistened with a strong solution of calcic chloride.

(*c*) The tips were slightly washed with pure water.

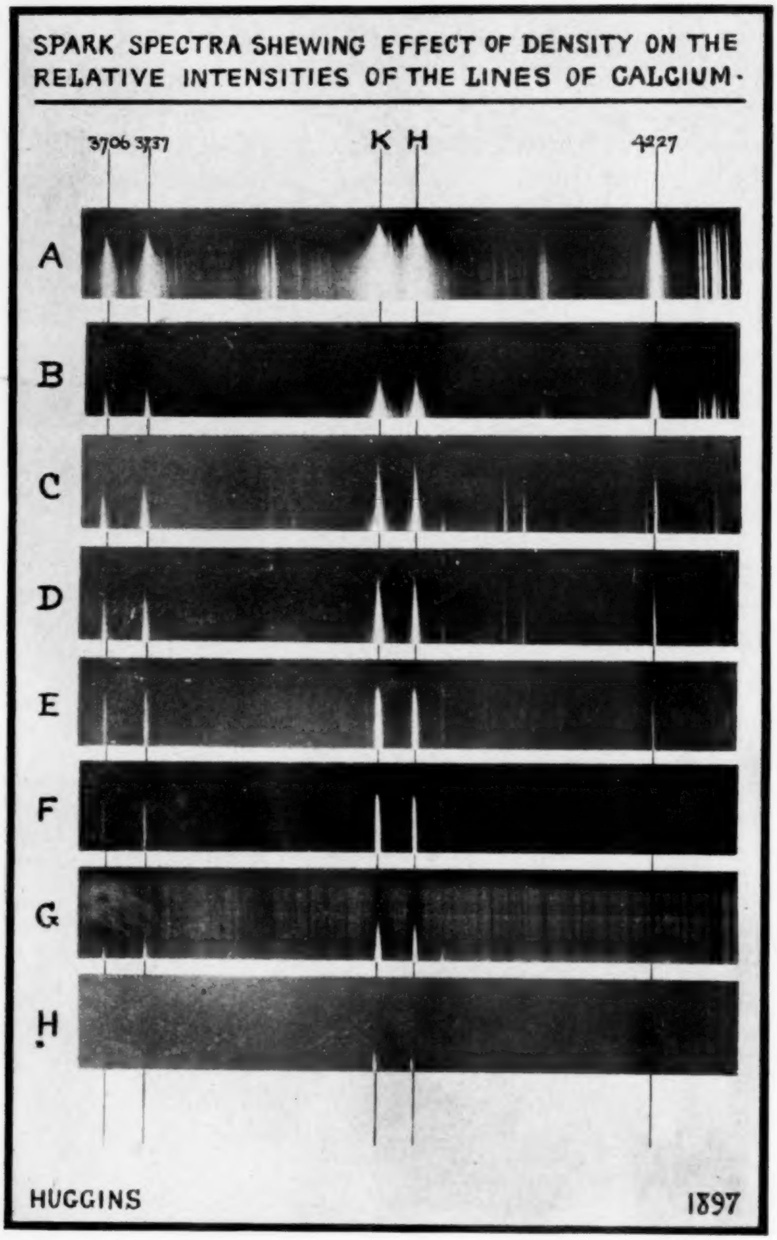
(*d*) The tips were again washed with pure water.

(*e*) The tips were then slightly moistened with a very weak solution made by adding a drop of the strong solution to two ounces of water.

Our expectations were completely confirmed. Under the conditions (*a*) of greatest density of the calcium vapor, when metallic calcium was employed, the blue line was as strong and possessed the same diffuse character as H and K.

As the density of calcium was reduced, the lines were not found to be equally enfeebled, but, on the contrary, the blue line and the greater number of the lines were increasingly reduced in intensity relatively to H and K, until at last with the twice washed electrodes (*d*) the spectrum was simplified to the con-

PLATE XIII.


SPARK SPECTRA SHEWING EFFECT OF DENSITY ON THE RELATIVE INTENSITIES OF THE LINES OF CALCIUM.
3706
3737
K
H
4227
A
B
C
D
E
F
G
H
HUGGINS
1897

dition usually existing in the prominences, in which H and K only are present.

We now proceed to a more precise statement of the changes of relative intensity as they are presented in the photographs which accompany this paper.

DESCRIPTION OF THE PHOTOGRAPHS ON PLATE XIII.

A. Photograph of the spark when both electrodes consist of metallic calcium. Here we have present doubtless the largest amount and greatest density of calcium vapor. The winged character of H and K, of the blue line, and of the pair more refrangible than H and K, is well seen, showing that this appearance comes out when the gas is dense. If the greater extension of the wings of H is allowed for, and the line H carefully distinguished from the fine lines close to it, it will be seen to possess very nearly the same strength, both as regards width and length, as the blue line at 4226.9. The strength of this blue line under this condition of density is about the same as that of the line at 3737, and rather greater than the line beyond at 3706.

B. Spark taken with one electrode only of metallic calcium, the other electrode being of platinum. In this case the effect of a smaller density of the calcium vapor is clearly shown in the greatly reduced wingedness of the lines. It will be remarked that the diminished density has had the greatest influence on the pair at 3737 and 3706; these lines are now much less strong than the blue line, which still holds its own, and remains about as strong as H and K. The lines of the more refrangible pair are no longer diffuse at the edges.

C. Spark taken between platinum electrodes moistened with a strong solution of calcium chloride. Here the effect of a smaller quantity of vapor begins to tell strongly upon the intensity of the blue line relatively to H and K. It may now be estimated at less than one-fourth of the intensity of H. At the same time H and K have almost completely lost their diffuse character, and have become thinner and more defined.

D. The electrodes as left in the former experiment were

slightly washed with pure water, leaving a trace only of calcium chloride. There is, as might be expected, an advance in the enfeeblement of the blue line and of the more refrangible pair, relatively to H and K.

E. The electrodes were again slightly washed with pure water, so that a still smaller trace of calcium chloride must have remained upon them. The enfeeblement of the blue line and of the pair has now become very great, while H and K, though thinner, remain strong.

F. The electrodes were once more washed with pure water, reducing still further the trace of calcium chloride which remained upon the platinum wires. The blue line has now practically disappeared, and the refrangible pair become very thin. The H and K lines have become thin and defined, as they usually present themselves in the prominences.

G. The electrodes remaining as they were left after the last experiment (F), the spark was taken upon a background consisting of a faint solar spectrum. The blue line has now completely disappeared, leaving H and K strong.

H. Once more the electrodes were washed, with the expectation of having removed completely the last remaining trace of calcium. To our surprise, when the photograph was developed, the lines H and K came out alone. The more refrangible pair had now faded out as well as the blue line. H and K were now thin, and extended but a short distance in the spectrum.

It must be remembered that the only condition which was varied during this set of experiments was the amount or density of the calcium vapor. The changes of relative intensity, and the modifications of the calcium spectrum produced thereby as shown in the succession of photographs on the plate, correspond closely to the behavior of calcium at different levels near the Sun's limb, and in the atmospheres of stars of different orders. There can remain no doubt that the true interpretation of the changes in appearance of the calcium lines in the celestial bodies is to be found in the different states of density of the celestial gases from which the lines are emitted or by which they are absorbed.

A similar set of experiments was made with iron electrodes. Precisely similar results as to the relative enfeeblement of the lines, as with calcium chloride on platinum electrodes, were obtained. Of course the iron lines were also present. As might be anticipated, in consequence of the simultaneous presence of the iron vapor, the lines of calcium were thinner than when platinum was used.

Outside the range of wave-lengths which could be conveniently given on the plate, far on in the ultra-violet, there is a pair of strong lines which behave very much as H and K. It remains visible in photograph H, when the pair at 3737 and 3706 have disappeared. This pair is situated at 3158.98 and 3179.45.

It is desirable to point out again that all the photographs on the plate and the far ultra-violet lines, were obtained with a spark of quite unusually small intensity, which was purposely made as little hot as possible, in order to emphasize the important fact that the determining condition of the spectral changes under discussion is not one of increase of temperature.

In the modifications of the calcium spectrum arising from variations in the relative intensities of the lines which have been discussed in this paper, and which correspond to those observed in the celestial bodies, there does not appear to us any reason for assuming, much less any direct evidence in favor of, a true dissociation of calcium, that is, of its resolution into chemically different kinds of matter.

It would be remarkable if, by decomposition through increase of temperature, a large number of lines of a spectrum should become relatively enfeebled, and that as the result of decomposition a spectrum should become simpler, and not as analogy would suggest, more complex.

It is of importance to keep in mind that the recent chemical use of the word *dissociation* is not equivalent to true decomposition, *i. e.*, to a resolution of the original substance into two or more chemically different kinds of matter. It may, and does often mean not more than a different arrangement of the parts of the molecule, while those parts are all chemically matter of the

same kind as the original molecule. As in the case of the resolution of a compound molecule of peroxide of nitrogen into two identical half molecules; or, in the separation of a molecule of elementary iodine into two half molecules or atoms of identical chemical characters. Such dissociations are well known, and are not of infrequent occurrence, and may, indeed, take place in connection with some of the spectral changes of a substance observed under different conditions. On the other hand, a true decomposition of a chemical element, that is, a breaking up of the molecule into simpler and quite other kinds of matter, though a notion familiar to chemists since Prout's time, and regarded as theoretically possible, is, as yet, unknown as a matter of fact.

CONCLUSIONS.

These experiments seem to us to furnish an adequate and consistent explanation of the behavior of the calcium lines at and near the Sun's limb. Near the photosphere, where the absorption mainly takes place by which the dark lines of the solar spectrum are formed, there would be, we should expect, a much greater density of calcium vapor than at a higher level, and we find the Fraunhofer line at 4226.9 strong but much less broad than H and K. The recent photograph of the reversing layer shows that the broad shading of H and K is not produced there, but probably, as Mr. Jewell concludes from his measures, lower down where the gas is still denser, which is in agreement with photograph *A* on the plate.

Higher up in the chromosphere, in the prominences, and possibly in the lower coronal regions, the decrease of the density of the gases composing them must be rapid, and the temperature gradient as determined by expansion must be rapid. We have clearly to do, in these regions, with calcium vapor in a rarer state, and except so far as the molecules may have carried up within themselves to some extent the higher heat of a lower level, or through imperfect transparency, the gases may have received heat from the Sun's radiation, it must be at a much lower temperature than near the photosphere. Now, the changes of the calcium spectrum which take place in these regions, are

those which correspond in our experiments to a very small amount of calcium vapor, and a spark of small intensity.

On account of the violent commotion which must exist through the strong convection currents at the Sun's limb, we should not be surprised to find some calcium vapor, notwithstanding its greater density, carried high up together with the lighter substances such as hydrogen and helium. Our experiments show how strongly the H and K lines may come out when a trace only of calcium vapor is present, and so, it seems to us, offer a possible explanation of the great height at which these lines may be sometimes recognized. At no very great distance from the surface of the Sun the gases must become too tenuous to give a visible spectrum, and it may well be that the brilliant radiations of even very rare calcium gas at H and K may show in our instruments for some distance after the hydrogen and the other light matter associated with it, have become too subtle to furnish a spectrum that we can detect.

The relative behavior of the lines of the calcium spectrum as they present themselves in the different orders of stellar spectra, when interpreted by the terrestrial experiments described in this paper, will throw important light on many of the important questions which are still pending in celestial physics. In forming conclusions as to the state of the stellar atmospheres from the different densities which may be indicated by the modifications of the calcium spectrum, it must be borne in mind that, as I have said elsewhere:

> "The conditions of the radiating photosphere and those of the gases above it, on which the character of the spectrum of the star depends, will be determined not alone by temperature, but also by the force of gravity in these regions; this force will be fixed by the star's mass and its stage of condensation, and will become greater as the star continues to condense."[1]

It may be, though on this point we have no sufficient data, that though the stars are built up of matter essentially similar to that of the Sun, the proportion of the different elements is not the same in stars which have condensed in parts of the heavens widely distant from each other, or at epochs greatly separated in time.

[1] Address, *Report Brit. Assoc. A. Science*, 1891, p. 15.

It does not seem desirable to discuss any of these questions at the present time, as we hope before long to offer some explanation of the, to some extent analogous, relative behavior of the lines of some other substances as observed in the Sun and stars.

Addendum.—The following letter from our friend Professor Liveing, which he has given us permission to publish, contains an account of early experiments on the spectrum of calcium, which not only support, by a different method of working, the conclusions of our paper, but also seem to show the possible occurrence of the line H, without the line K. In our experiments, as will be seen in the spectra on the plate, both lines are alway present, while the line K is stronger and longer than the line H ; which agrees with the photographs of the prominences taken by Hale, and by Deslandres.

"I have been looking up some observations of Dewar's and mine on the H and K lines of calcium made in 1879. We found that when we used, for the arc, carbon poles which had been heated for two days in chlorine to remove metals, the calcium lines were not at first visible in the arc, but after a time H was seen alone and not strong, after a further time K was seen, and then other calcium lines came out. No doubt the calcium had been pretty well removed from the carbon rods to some depth, but not entirely from the interior, so that as the carbon burnt away in the arc the calcium in the interior became manifest.

"Again we found that when we used a perforated pole and passed a stream of hydrogen into the arc through it, H and K could be both entirely obliterated; but by then reducing the current of gas they gradually reappeared, and H always came out first and afterwards K, and H remained stronger than K until they had both become strong, and had resumed their ordinary appearance. This was seen many times.

"Both observations seem to me to confirm your conclusions. In the latter case the stream of hydrogen diluted the calcium vapor, and the degree of dilution was controlled by the rate at which the gas was introduced. The mass of gas passing was too small to reduce the temperature by any considerable amount, or even, I should think, by any sensible amount.

"We found also that metallic lithium introduced into the arc produced effects similar to those produced by hydrogen, that is, it reduced very much the strength of the H and K lines. If more than a very minute piece of lithium were introduced the arc was invariably broken, so that we did not notice the complete obliteration of H and K with the lithium. The reduction of the strength of H and K in this case I attribute to the dilution of the calcium vapor by that of lithium."

THE NEW PHOTOGRAPHIC CORRECTING LENS OF THE EMERSON McMILLIN OBSERVATORY.

By H. C. LORD.

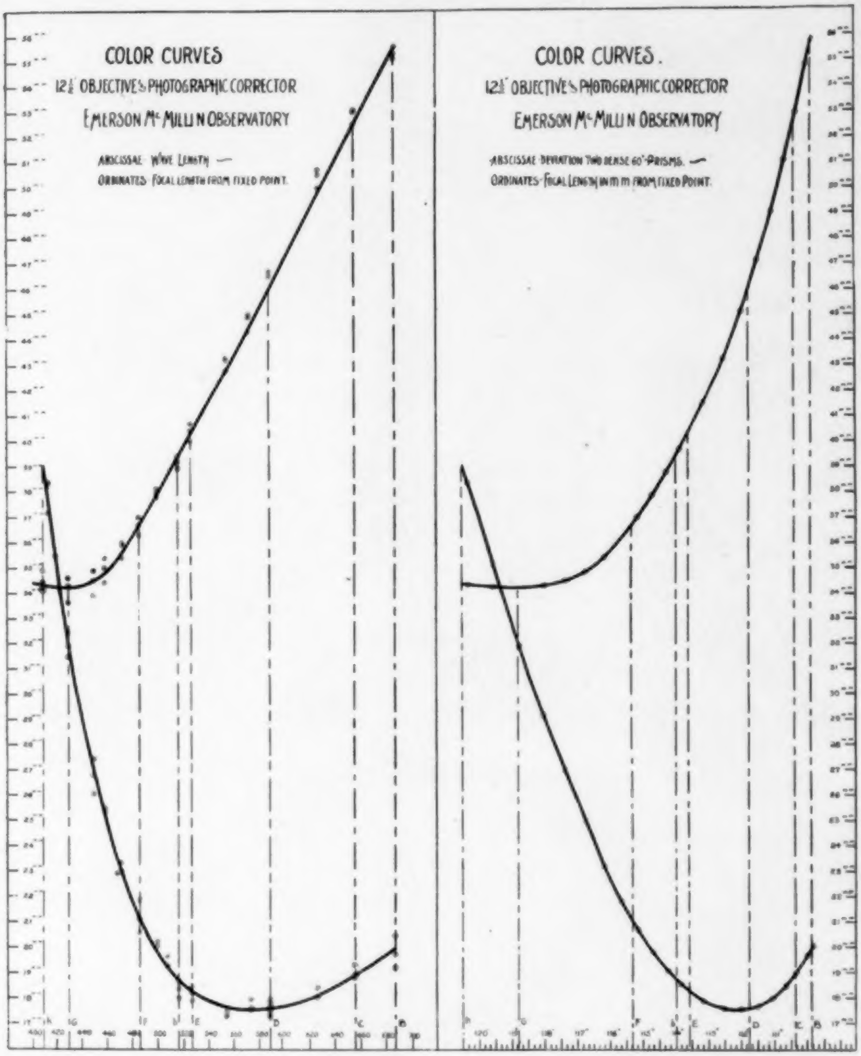
IN a paper published in the ASTROPHYSICAL JOURNAL for February 1895, Professor James E. Keeler describes a lens, which, placed a short distance above the slit of a compound star spectroscope, so alters the color curve of the large objective, as to render it flat in the photographic portion of the spectrum. He there calls attention to the fact that if this lens be compound the correction can be secured without materially altering the focus for any selected ray. The use of a single lens of this nature is described by Professor H. F. Newall in a paper on the Bruce spectroscope published in the *Monthly Notices of the Royal Astronomical Society* for January 1896; but, so far as I am aware, the compound lens has never before been tried in actual practice. I trust, therefore, that the description of such a lens, in use at the Emerson McMillin Observatory, may be of interest to the readers of the ASTROPHYSICAL JOURNAL.

The spectroscope is fully described by the author in Vol. 4, No. 1 of this JOURNAL. As soon as possible after the completion of the Observatory, the color curve of the 12½-inch objective was determined, observations being made on three different nights. This curve, together with the plotted observations, is given in a plate which accompanies this article. Numerous attempts were made to photograph stellar spectra in the neighborhood of $H\gamma$ without success, as it was found absolutely impossible with the slit set at the focus for this line, to tell whether or no the star's image was within the jaws of the slit. Professor Keeler, in the article above quoted, has called attention to the fact that this difficulty must always be experienced when attempting to photograph with a visually corrected objective. In order to test the accuracy of the following, the camera of the spectroscope was replaced by the observing telescope, and

the star brought on the slit by means of the device for following. In almost every case the star's spectrum was not seen in the observing telescope. In this way it was found that the limit at which accurate following was possible was but slightly to the violet side of the $H\beta$ line. The spectra were, moreover, excessively short and even a slight change of focus caused the point of maximum density of the spectrogram to shift by a considerable amount. It was decided, therefore, to provide the telescope with a compound correcting lens. Mr. Emerson McMillin, the founder of the Observatory, generously furnished the necessary funds. The color curve of the large objective was sent to Mr. Brashear and from it the curves of the lenses of the compound correcting lens were computed. The corrector is 76^{mm} in diameter and is fitted to a long brass tube which screws into a sleeve fastened to the inside of the breech piece of the telescope tube, holding the lens 1011^{mm} above the uncorrected focus for $H\beta$. The method of support is most excellent. It is very rigid and can be removed inside of two minutes. As soon as it was received the color curve was determined and is given, together with the plotted observations, in Plate XIV. I give below a table which gives both for the corrected and uncorrected objective the distance of the foci for the several rays, from a point one meter above the uncorrected focus for $H\beta$. In Plate XIV are given two sets of curves. The first are the two color curves with ordinates focal lengths from a fixed point, and abscissæ wave-lengths. The advantages of the new lens when used with a prism spectroscope are best brought out by the second set of curves. Here the ordinates are as before, but the abscissæ are the deviations produced by two dense $60°$ prisms, such as are in use with this spectroscope. The scale is so taken that the distance from B to *h* is nearly the same in both sets of curves. Since photographic action ends slightly below $H\beta$, this combination may be called practically achromatic for the photographic portion of the spectrum.

The effect of the corrector on the ease of following is very marked. With the slit set at the corrected focus for $H\gamma$, and my

PLATE XIV.


COLOR CURVES
$12\frac{1}{4}$" OBJECTIVE & PHOTOGRAPHIC CORRECTOR
EMERSON MC MILLIN OBSERVATORY
ABSCISSAE WAVE LENGTH —
ORDINATES FOCAL LENGTH FROM FIXED POINT.
COLOR CURVES.
$12\frac{1}{4}$" OBJECTIVE & PHOTOGRAPHIC CORRECTOR
EMERSON MC MILLIN OBSERVATORY
ABSCISSAE DEVIATION TWO DENSE 60° PRISMS. —
ORDINATES FOCAL LENGTH IN M M FROM FIXED POINT.

assistant stationed at the observing telescope of the spectroscope, I watched the star's image in the device for following, which is similar to that in use at Potsdam, and by means of the declination slow motion, threw the star off and on the slit, calling out to him when the spectrum should be seen. Without telling me he noted the times it appeared. Out of twenty trials I failed but once. By removing the eyepiece of the following telescope (a plan suggested by Professor Keeler) following is very easy; the slighest pressure on the supporting rods of the spectroscope is at once detected. With Dr. Huggins's reflecting slit following is still more certain. The star's image is very peculiar; a green central point is surrounded by a bright halo of red rays and no difficulty has been experienced in bisecting the central green point with the slit.

In order to measure the field of the new lens the slit was removed from the collimator and a rough, homemade photochronograph[1] put in its place. This was connected with the standard clock of the Observatory and an exposure of about one-tenth of a second given every second as the star was allowed to trail across the plate. When developed the plate showed a series of small dots or circles, which, when examined under the microscope, appeared sharp and distinct over about $5^{mm}.3$, after which they rapidly degenerated into more and more elongated ellipses, whose major axes pointed to the center of the field. This distance gives a field of about four minutes of arc, much larger than is needed in spectroscopic work.

As a further check upon the color curve a photograph of α Lyræ was taken with the slit set parallel to an hour circle, in order to secure a spectrogram which should be as narrow as possible. An exposure of 15^m showed $H\beta$, $H\gamma$, and $H\delta$, and both H and K, but the last two faintly. From $H\beta$ to $H\delta$ the spectrum is of almost uniform density but of course greatly over-exposed. The lower limit of the photograph reaches nearly to the little *b*'s. Its width is at $H\beta$ $0.^{mm}132$, at $H\gamma$ $0.^{mm}068$, and at $H\delta$

[1] "The Photochronograph and its Applications," *Publications Georgetown College Observatory.*

$0.^{mm}064$ approximately. Possibly the best testimonial to the advantages of this lens is shown in Plate XV. The two spectra are I, the Sun, and II, that of Arcturus. The latter extends from below $H\beta$ to above $H\delta$. It should be stated that the relative densities of the different portions of the photograph of Arcturus have been controlled by what might be called a paint brush method of development, my object being to show what extent of spectrum could be secured on one plate. I should also state that the enlargement was made with a home-made piece of apparatus and of course the sharpness of the lines in the original negative suffered more or less.

TABLE I.

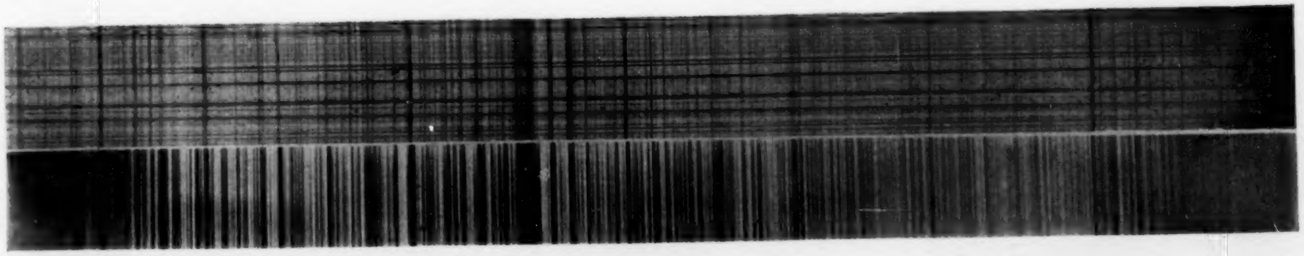
Wave-length	Distances of Foci from a point 1^m above uncorrected Focus for $H\beta$	
	Uncorrected Lens	Lens with Corrector
B	$998^{mm}.8$	$1034^{mm}.2$
C	997 .8	1031 .1
6300	997 .1	1028 .6
D	996 .5	1024 .6
5500	996 .6	1021 .0
E	997 .1	1018 .9
b's	997 .6	1018 .0
$H\beta$	1000 .0	1014 .3
4600	1003 .9	1013 .4
$H\gamma$	1010 .7	1012 .8
h	1018 .0	1012 .9

EMERSON MCMILLIN OBSERVATORY,
June 16th, 1897.

PLATE XV.

I.

II.



Photographed by Prof. H. C. Lord.

I. SPECTRUM OF THE SUN.

II. SPECTRUM OF ARCTURUS.

ON THE LEVEL OF SUN-SPOTS AND THE CAUSE OF THEIR DARKNESS.

By A. Riccò.

In the October 1896 number of the Astrophysical Journal Professor E. B. Frost has again called in question the theory of Wilson on the level of Sun-spots. He presents important physical considerations, and shows that spot statistics obtained from drawings or photographs have not given definitive results in demonstration of the fact that the spots are cavities.

These considerations have led me to bring together statistics derived from a series of drawings of Sun-spots made by the method of projection in the years 1880 to 1890 at Palermo, with a refractor of $0^m.25$ aperture, and at Catania in 1892 with a refractor of $0^m.33$ aperture. The diameter of the projected solar disk was in all cases $0^m.57$, large enough to show satisfactorily the principal details of the spots.

In bringing together the statistics I have omitted the drawings of the first year, 1880, since it is probable that at that time I had not acquired sufficient skill to represent the Sun-spots with the necessary exactness. The break in 1891 was caused by the fact that the Catania refractor had not then been mounted. I might have filled this gap by using the observations continued by Professor Mascari at Palermo, but I have preferred not to do this in order to render the series as nearly as possible homogeneous.

In these eleven years the number of days of observation was 3451 and the number of drawings of spots 17,436 (excluding the pores), of which 3324 represent different spots. Rigorously excluding all spots not perfectly regular in form, *i. e.*, round and symmetrical when near the center of the Sun's disk, there remain only 185 drawings suitable for the study of the question. The following table gives the data regarding the width of the penumbra on the side toward the Sun's limb and on the opposite side:

Year	Penumbra of regular spots		
	Wider toward the limb	Narrower toward the limb	Symmetrical
1881	28	0	9
1882	16	7	2
1883	19	2	2
1884	22	5	4
1885	15	1	6
1886	10	0	6
1887	7	0	0
1888	0	1	0
1889	1	0	3
1890	2	1	1
1891	..	..	..
1892	11	1	3
Total	131	18	36

It is seen that in each of the eleven years (except 1888, when there was but one regular spot), the number of spots near the limb whose projected form gave a result conforming to the theory of Wilson is much greater than the number of contrary or uncertain cases. From the entire eleven years we have the proportion of cases favorable, unfavorable, and neutral to the theory of Wilson:

favorable : *unfavorable* : *neutral* :: 7.3 : 1 : 2

Greater weight must be given to the still more significant cases of spots near the Sun's limb, the penumbra of which, conforming to the appearance of a cavity seen in perspective, is invisible on the side opposite the limb. I have found twenty-three cases of this sort in the eleven years, and only one contradictory case.

It must be added that if it is admitted that the umbra and the penumbra of spots are like dark clouds elevated above the photosphere, it is necessary to suppose that they are enveloped by the tongues and flames rising from the photosphere, which are frequently observed across the spots, where they diffuse into the nucleus; and the best drawings of large spots made by the ablest observers, distinguished astronomers, and skilful draughts-

men (for example, the "typical spot" of Langley) would be wholly incomprehensible.

It must, therefore, be admitted that the spots are cavities, *i. e.*, breaks or openings in the photospheric layer; but it is very difficult to say what is the condition of the materials which they contain and which are the cause of the darkness of the umbra and penumbra. In fact: (1) the thermal, visible, and actinic radiations of the spots are in general less intense than those of the photosphere; but (2) these radiations have almost the same intensity when the spots are near the center of the Sun as when they are near the limb; (3) the temperature of the Sun increases rapidly in passing toward the interior layers; and (4) the outlines of the penumbra and the umbra are not diffuse, but are clearly defined. These reasons (2, 3, 4) do not permit the conclusion that in the spots there is a substance which absorbs on account of its lower temperature. On the contrary, (3) would rather lead us to suppose that in the spots there is a substance but slightly luminous on account of its excessively high temperature; but in this case, as Mr. Evershed[1] has very properly pointed out, it must also be very transparent; hence the radiations of the internal layers would be integrated and the obscurity should disappear, or rather one should be able to see the radiations of the photosphere on the opposite side of the Sun.

To avoid this difficulty Mr. Evershed has suggested the very ingenious hypothesis that on account of the high temperature of the materials contained in the spots, the radiations of short wave-length must be increased while those of greater wave-length are diminished, so that the spots should be particularly rich in ultra-violet light, *i. e.*, dark to our eyes. But they must have a photographic action, while as a matter of fact the penumbra and umbra of spots are dark in solar photographs. To avoid this difficulty we must suppose with Mr. Evershed that the absorption exercised on the radiations of short period by the terrestrial atmosphere (as is demonstrated by the red color of the Sun at the horizon), and also perhaps, as it seems to me,

[1] This JOURNAL, April 1897, p. 296.

by the solar atmosphere (demonstrated by the dark reddish color of the Sun's limb), is strong enough to suppress the actinic action of the nuclei of Sun-spots.

In considering the relations which are known to exist between radiations of short period and electrical phenomena, the suggestion of Mr. Evershed that the radiations peculiar to Sun-spots, although invisible, may be the cause of the perturbations of terrestrial magnetism which coincide with the presence of large spots on the Sun's disk, seems to me of the greatest importance. Such a coincidence cannot be denied, as I have myself pointed out.[1]

In conclusion, considering the importance of the problem of the constitution of Sun-spots and the difficulties which stand in the way of its solution, we may echo the wish expressed by Mr. Evershed that the radiations of Sun-spots may be studied in a more complete manner. It is to be hoped that Professor Hale, who has long intended to make a detailed and complete study of spots, may be able to carry out his project now that he has at his disposal the admirable instrumental equipment of the Yerkes Observatory.

CATANIA OBSERVATORY,
May 5, 1897.

[1] *C. R.*, October 17, 1892; *Mem. Spettr. Ital.*, **21**, 153.

INVESTIGATION OF THE VIOLET PART OF SOME METALLIC SPECTRA WHICH CONTAIN MANY LINES.[1]

By O. Lohse.

While investigating the spectra of a considerable number of metals within the region λ 4000–λ 4600 by means of a spectrograph, I found several that were remarkable on account of the great number of lines which they contained. They were mainly spectra which are only imperfectly known, at least in the region which I have studied, and I believe therefore that the following communication concerning the wave-lengths of the lines will not be without interest. It is my intention to repeat the investigation of the spectra of these metals with the aid of a concave grating, and to extend it to other parts of the spectrum; hence I regard the observations here given as only preliminary.

The spectrograph which has been used in this investigation has a direct-vision prism filled with ethyl cinnamate and closed at the ends by plates which are perpendicular to the axis. On one of these plates a total-reflecting prism is so placed that the rays which come from the collimator and fall on the upper half of the fluid prism are reflected back through the lower half. The dispersion is therefore very considerable, and the linear extent of the spectrum, between the wave-lengths above mentioned, is 180^{mm}. The collimator objective has a focal length of about $\frac{3}{4}^{m}$ and is achromatized for the chemically active rays. A single plano-convex lens of 1^{m} focal length was used as a camera objective, as I found that an achromatic lens gave no better result than the former and absorbed more light. Both objectives have an aperture of 40^{mm}.

The apparatus, which is mounted on a heavy cast-iron pillar, was made in the well-known shops of O. Toepfer, in Potsdam;

[1] "Untersuchung des violetten Theils einiger linienreicher Metallspectra." *Sitz. d. K. Akad. d. W. Berlin*, **12**, 179–197, 1897.

it is satisfactory in all respects, and observations are made with remarkable ease and certainty. The slit, which has platinum-iridium jaws, is vertical. It has an attachment which can be moved up and down, so that as many as seven narrow spectra can be photographed one over another on the same plate, though the whole length of the slit can also be used. The camera to which the plate-holder is secured is provided with a bellows; it moves along a planed track of cast-iron, furnished with a graduated scale. Since my observations were confined to spark-spectra, for reasons which I will refer to further on, I adopted an arrangement by which a condenser, made by me of a combination of lenses and a silvered-glass mirror, and intended to increase the light from the spark, could be moved up and down in front of the slit, together with the horizontal electrodes.

The electric current necessary to operate a large induction coil 39^{cm} long was generated by a dynamo, which was driven by a gas-engine. Since the current strength required was only a small fraction of that furnished by the dynamo, and it was necessary to protect the coil from the effects of too great a current, a sufficiently high resistance was always interposed and a shunt circuit used for the coil, the strength of which could be reduced still further by increasing the same resistance. With the aid of this arrangement a very uniform stream of sparks was obtained.

In photographing with this instrument the greatest difficulties naturally arose from the influence of heat on the refractive power of the ethyl cinnamate, an influence which is so great that perceptible effects are caused by changes of temperature that do not admit of measurement with even the more delicate thermometric instruments. This caused with rough contrasts inferior definition of the lines; in general only continuous wandering of the spectrum was produced, which was prohibitive of long exposures. A further result of the wandering of the spectrum was that the distance of the focal-plane from the objective was altered, particularly in oblique positions of the photographic plate. It was not until I had taken account of this effect and

focused anew for each series of experiments that I succeeded in obtaining uniform results.

On account of the uninterrupted motion of the image I was further compelled, in order to obtain reliable points of reference, to photograph at the same time the spectrum of iron, the lines of which are easily identified in the solar spectrum. This was very easily done by always taking iron for one of the two electrodes, in which practice it only very rarely happened that the superposition of lines necessitated the taking of additional photographs with electrodes of the same kind.

In all photographs of metallic spectra the exposures were seventy seconds, with a slit-width of $0^{mm}.03$.

The photographs were measured with the aid of a simple apparatus, which consisted essentially of a microscope with micrometer eyepiece, mounted on a table. With this arrangement it was possible to measure only a short range of the spectrum, corresponding to the circumscribed field of view of the microscope.

The micrometer screw was cut by Herr Toepfer, and its periodic errors were so small that they were without perceptible influence on the accuracy which was aimed at in the measurement of wave-lengths; hence the direct readings of the micrometer could be used.

All measures were referred to the solar spectrum, on a particularly good photograph of which I had indicated by figures the normal lines accurately determined by Rowland. Forty-seven of these normal lines were used in the region between λ 4000 and λ 4600.

This photograph was fastened once for all to the glass stage of the measuring apparatus, and the metallic spectrum was laid upon it, so that the two films were in contact, and the lines as nearly as possible in coincidence.

The latter adjustment was effected with the aid of the iron lines, which are easily recognized in the solar spectrum, through their regular arrangement, when the iron spectrum is held opposite to it. An exact coincidence throughout the whole spectrum

could not, however, be attained, on account of the changes of refraction and dispersion caused by variations of temperature. There were no two plates which were quite the same in this respect. In making the measurements care was taken that at least one of the normal lines above mentioned was in the field of the microscope. Its position was generally determined by three, sometimes by more, settings; then several iron lines were measured, in both the solar and the metallic spectrum, in order to determine the fortuitous relative position of the two spectra, after which the measurement of all the lines in the field of view was begun, with three settings on each line. The displacement of the metallic spectrum relatively to the solar spectrum could in most cases be regarded as constant for the region included in the field of view; in only a few photographs, which were taken at a temperature widely different from that at the time of taking the solar spectrum, were the differences of dispersion found to be so considerable that the displacement had to be regarded as variable even within the field of view.

The relation between the micrometer revolutions and wave-lengths was found by measuring in succession the greater part of the normal lines in the solar spectrum, in such manner that the measurements were connected, each new series beginning with the last line of the preceding series. As the field of the microscope was comparatively small, thirty series were necessary. The entire length of the spectrum was then subdivided into smaller parts, for each of which the necessary reduction factors were determined by the method of least squares. It was found that at least seven subdivisions were necessary in order to find the wave-lengths with the requisite precision when terms of the second order were taken into account. After the completion of the computations a number of tables were made, by which the change from revolutions to wave-lengths was much facilitated, so that the labor of determining the wave-lengths of the metallic lines was not so great as one might be led to suppose from the foregoing account of the process.

The uncertainty of the wave-lengths deduced from my

measurements does not ordinarily much exceed 0.1 Ångström's unit; the existence of a greater error is probable only when the corresponding line is extremely faint, so that it could not be seen without difficulty, or when its breadth was unusually great.

In cases where comparisons were possible, I have found that my results agree satisfactorily with other recent observations, and in particular with the tables of Rowland, published in the Astrophysical Journal, where special prominence is given to those lines of the solar spectrum which coincide with metallic lines. On the average, some 20 per cent. of the lines of cerium, lanthanum, yttrium, zirconium and vanadium which I have observed are found in these tables, so that a valuable check on the work was thus obtained.

The reason for limiting the investigation to the region $\lambda\,4000$--$\lambda\,4600$ was, originally, that it seemed chiefly desirable to learn the positions of metallic lines in the violet part of the spectrum where the greatest photographic activity is manifested, and thus to obtain data which would be useful for comparison with spectographic observation of the heavenly bodies. In the course of the work I remarked that the investigation of a large number of substances at the same time, within the same limited region of the spectrum, has certain advantages, in that it enables one to immediately recognize accidental and unexpected impurities. As a proof of the identity of lines I constructed a table having wave-lengths as arguments, with intervals of 0.1 tenth-meter. The observed metallic lines were gradually inserted in this table, by which process it was at once recognized when any particular place was already occupied by a line of another metal. In places where a large collection of figures occurred, which did not differ from one another by more than the probable error of observation, it was certain that the existence of some element as an impurity in a number of metals was indicated. The identification of this element was effected with certainty by comparing the tabulated intensities of the lines.

In recent times the investigation of metallic spectra by means of the induction spark has fallen somewhat into the back-

ground, and the principal spectroscopists have used the electric arc for vaporizing metals. The cause of this fact appears to be that, in consequence of the variations of temperature with one and the same metal in the arc, a greater number of lines make their appearance than in the spark spectrum. But it is precisely this difference which offers much that is of interest, and renders a continuation of the investigation of spark spectra desirable. I have, therefore, adopted the latter method. The induction spark has the further advantage over the electric arc that it does not give rise to an accompanying continuous spectrum, and that hence the lines appear on a dark ground; and, finally, the consumption of the material of which the electrodes are composed is very slight—a consideration of some importance where so many rare metals are concerned.

At first I did not consider that it would be possible to use other than metallic electrodes, but as the investigation would thereby have been limited to a small number of metals, I finally used compounds also. The chlorides or nitrates soluble in water were always chosen. Sticks of charcoal (following Bunsen) were saturated with the solution after they had been heated to dull redness. These sticks served as electrodes and proved to be very good. At the great heat of the electric spark the compounds were reduced by the carbon, furnishing the requisite quantity of metallic vapor in a state of comparative purity and in an economical manner. The existence of other substances was easily recognized by the constant recurrence of certain lines.

With regard to the following tables it is to be mentioned that the intensities were estimated on a scale of 10, so far as this was possible without the aid of other methods. The intensity 1 indicates a very weak line, 10 a very strong line. Smaller steps than those given by this scale of intensity could be observed, particularly toward its lower limit, but, to avoid carrying the system too far, they were not taken into account.

When *n* is printed in place of the intensity, or as a subscript to figures expressing intensity, it is intended to signify that the corresponding line is not sharply defined.

In the case of wave-lengths which are given in demonstration of accidental coincidence or of indentity, the intensities according to my estimates are printed as subscripts to the customary abbreviations of the names of the metals; subscripts to other wave-lengths also indicate intensities.

CERIUM; LANTHANUM; DIDYMIUM.

Since it was not to be expected that metallic cerium could be obtained in pieces sufficiently large to serve as electrodes, I used the chloride according to the method already described. This and all other preparations were furnished by Dr. Schuchardt, of Görlitz, Silesia. I suspected contamination by lanthanum and didymium, and therefore photographed the spectra of these two metals on the same plate with that of cerium. The generally strong lines of lanthanum were also noticed in the spectra of cerium and didymium. Although all the lanthanum and cerium lines were eliminated from the cerium spectrum during the process of measurement, there still remained some 400 cerium lines in the small range of spectrum which was measured. Of all metals, therefore, cerium may be regarded as that possessing the greatest number of spectral lines in the violet. The spark spectrum of iron within the same region on my plates showed about 130 lines.

SPECTRUM OF CERIUM.

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4016.01	2		4027.82	3	
4017.06	1		4028.53	4	
4017.69	2		4030.47	3	
4019.18	3		4031.46	4	
4020.22	2		4032.74	1	
4020.65	1		4037.83	3	
4020.97	1		4038.41	3	
4021.43	1		4040.03	1	
4021.90	1		4040.89	5	
4022.40	3		4041.46	1	
4024.62	4	4024.66Zr_6	4042.29	1	
4025.24	2	4025.14Zr_4	4042.71	4	

SPECTRUM OF CERIUM—*continued*.

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4042.90	2		4083.78	2	
4045.40	2	4045.92 Zr_6	4085.39	4	
4045.97	5		4086.59	1	
4046.49	3		4087.48	3	
4047.46	1		4087.66	1	
4049.19	1		4088.76	1	
4050.99	1		4089.02	3	
4051.54	2		4089.91	2	
4052.16	2		4090.63	2	4090.68 Zr_6
4053.65	3	4055.16 Zr_5	4091.11	2	
4055.13	3		4092.25	1	
4055.25	1		4092.86	2	
4056.38	1		4094.11	2	
4057.00	2		4099.13	2	
4058.37	1		4099.94	1	
4058.91	1		4101.93	4	
4060.05	1		4102.62	2	4102.56 Y_5
4060.60	2		4104.59	1	
4060.83	1		4105.16	3	
4061.23	2	4061.28 Di_3	4106.28	1	
4062.36	3		4107.02	3	
4063.08	2		4107.55	4	
4064.06	1		4107.95	1	
4065.08	1		4108.41	1	
4065.16	1		4108.84	1	
4066.69	1		4110.53	3	
4067.43	3		4111.01	1	
4067.91	1		4111.51	3	
4068.15	1		4112.07	1	
4068.60	2		4113.86	2	
4068.96	4		4114.22	1	
4070.24	3		4115.48	4	
4071.00	1		4117.10	3	
4071.22	1	4071.26 Zr_3	4117.40	2	
4071.89	6		4117.70	3	
4073.07	2		4118.24	6	
4073.60	6		4119.13	3	
4073.87	3		4119.94	7	
4074.28	1		4120.94	4	
4074.80	1		4123.57	4	4123.46 La_{10}
4075.26	1	4075.14 Zr_4	4123.95	7	
4075.93	9		4124.88	4	
4076.42	3		4125.55	1	
4077.61	5	4077.58 La_9	4125.88	1	
4078.86	2		4126.74	1	
4079.46	1		4127.44	6	
4079.89	2n		4127.81	3	
4080.64	2		4128.13	2	
4081.38	5		4128.44	3	4128.49 Y_4
4083.38	6		4129.29	1	
4083.64	2		4129.84	1	

SPECTRUM OF CERIUM—*continued.*

Wave length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4130.81	4	4130.85 Ba_{10}	4187.41	4	
4131.26	4		4190.74	2	
4132.45	1		4191.17	1	
4132.81	1	4132.91 Th_4	4193.37	7	
4133.95	8		4194.05	3	
4135.55	4		4195.07	2	4195.00 Zr_8
4136.05	1		4196.00	1	
4137.04	2		4196.54	5	
4137.73	9		4197.79	2	
4138.26	3		4198.18	2	
4138.55	3		4198.87	7	
4139.65	1		4199.33	1	4199.34 Zr_6
4140.14	1		4201.53	3	
4141.00	1	4140.22 Zr_3	4203.18	5	
4142.63	5^n		4205.07	1	
4143.07	3		4205.37	1	
4143.40	1	4143.01 Y_4	4206.09	1	
4143.67	1		4208.64	1	
4144.12	3		4209.65	2	
4144.75	3		4213.30	1	
4145.26	5		4214.31	2	
4146.50	3		4222.88	6	
4148.45	1		4224.21	1	
4149.16	3		4224.85	1	
4150.15	9		4228.02	4	
4151.14	4		4228.57	1	
4152.22	8	4151.18 Zr_8	4230.45	1	
4153.38	2_n		4232.04	3	
4154 35	1		4232.27	1	
4155.56	1		4232.60	1	
4155.75	2		4234.44	2	
4159 22	4		4235.01	1	4234.90 Zr_4
4160.28	2		4236.51	1	
4161.30	2		4238.11	1	
4161.98	1		4240.14	5	
4162.81	1	4161.98 Sr_{10}	4242.30	1	
4163.67	4	4162.88 Th_2	4243.00	4	
4165.73	7		4244.05	1	
4166.35	1		4246.18	4	
4166.77	1		4246.99	3	
4167 00	1	Not separated	4248.26	1	
4167.91	4		4248.93	5	
4169.95	5		4251.92	1	Not separated
4172.22	2		4252.08	1	
4174.53	2		4253.60	3	
4175.34	1		4254.99	1	
4176.12	1		4255.97	3	
4176.78	3		4256.30	2	
4179.22	3		4257.37	1	
4185.46	3		4258.97		Band
4186.70	8		4259.93	1	

SPECTRUM OF CERIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave length tenth meters	Intensity	
4261.34	1		4332.98	2	
4264.08	1		4335.16	1	
4264.55	2		4335.81	1	
4266.10	1		4336.54	3	
4267.34	1		4338.08	5	
4268.03	1		4339.06	1	
4268.49	1		4339.66	3	
4269.44	2		4340.97	1	
4270.35	3		4342.52	1	4342.50 Zr_2
4270.86	3		4342.86	1	
4271.92	4		4344.24	1	
4273.62	2		4344.71	1	
4274.68	1	4273.68 Zr_6	4345.23	1	
4275.63	3		4346.27	2	
4278.42	1		4346.81	1	
4279.02	2		4347.99	1	
4280.29	2		4350.12	4	
4281.12	2_n		4350.78	1	
4283.67	1		4351.60	1	
4284.60	1		4352.36	1	
4285.53	4		4352.99	4	
4288.85	2		4353.71	2	
4289.66	2		4355.72	1	
4290.16	6		4357.18	1_n	
4292.93	3		4358.34	1_n	
4294.83	1		4359.39	1	
4296.90	8		4360.54	1	
4299.55	4		4360.74	1	
4300.50	4		4361.94	2	
4302.81	2		4363.75	1	
4303.77	2		4364.43	1	
4304.45	1		4364.91	4	
4304.97	1		4367.24	1	
4305.31	3		4367.75	1	
4306.95	5		4368.48	1	
4308.08	5		4369.51	1	
4309.93	4		4370.91	1	
4311.01	2		4372.61	1	
4311.95	2		4374.01	3	
4313.45	1		4376.03	3	
4314.84	1		4380.30	1	
4315.75	1		4382.09	1	Not separated
4317.59	2	4317.47 Zr_5	4382.38	5	
4318.29	1		4385.93	1	
4319.34	1		4387.04	5	
4321.04	4		4388.24	2	
4325.07	2		4390.55	2	
4327.11	1		4391.93	5	
4328.23	1		4393.42	2	
4330.74	2		4395.04	2	
4332.07	2		4396.28	1	

SPECTRUM OF CERIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4396.87	1		4477 48	1	
4397.52	1		4479.67	3	
4398.19	1		4483.66	1	
4399.00	2		4484.16	4	
4399.48	4		4485.08	2	
4400.86	1		4485.80	1	4485.76 Zr_s
4401.09	2		4487.20	4	
4403.62	1	4403.63 Zr_s	4494.49	2	
4405.65	1		4496.00	1	
4407.58	2		4496.50	2	
4409.11	2		4498.11	2	
4410.90	5		4500.57	1	
4412.17	n		4501.99	1	
4413.35	1		4508.29	1	
4414.02	1		4509.38	1	
4417.08	2		4510.32	1	
4418.99	5		4511.11	1	
4422.77	1	4422.76 Y_s	4511.82	1	
4423.84	2		4515.97	1	
4424.53	2		4519.73	1	
4427.28	2		4523.23	4	
4428.08	2		4527.30	4	
4428.65	2		4528.71	5	
4429.44	4		4532.73	1	
4433.01	1		4537.16	1	
4434.01	1		4538.21	1	
4434.58	1		4539.33	1	
4441.97	1		4539.99	4	
4443.98	2		4545.21	1	
4444.70	4		4549.90	1	
4444.95	5		4550.58	1	
4449.59	5		4551.54	2	
4450.94	4		4555.67	1	4555.63 Zr_s
4452.51	1		4560.45	4	
4458.03	1		4561.20	2	
4460.52	6		4562.57	5	
4461.50	3	4461.44 Zr_s	4566.04	2	
4463.77	3		4579.39	1	
4465.03	2		4582.65	2	
4465.78	1	4465.73 Th_s	4591.35	1	
4466.93	1		4594.06	4	
4467.85	2_n		4597.30	1	
4471.55	6		4606.54	2	
4473.04	3		4624.84	2	
4475.01	1		4628.10	5	

SPECTRUM OF LANTHANUM.

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4015.44	1		4280.45	2	
4023.79	1		4282.23	1	4282.33 Zr_6
4026.08	2		4287.12	7	
4031.88	6		4296.25	6	
4036.78	1	4036.70 Th_2	4300.62	2	4300.50 Ce_4
4037.44	1		4302.70	1	4302.67 Ca_9
4043.11	8		4316.12	1	
4046.01	3	4045.92 Zr_6	4318.90	1	4318.87 Ca_8
4050.27	5		4334.08	9	
4060.51	1		4335.27	4	
4064.99	1		4354.64	5	
4065.82	1		4363.33	1	4363.32 Ce_2
4067.61	5		4378.04	2	
4076.97	2		4383.70	6	
4077.58	9		4385.45	2	
4077.96	2	4077.91 Sr_{10}	4411.55	1n	
4079.47	1		4419.42	1	
4086.97	10		4423.42	1	
4089.85	1	4089.91 Ce_3	4424.18	1	
4099.73	5		4425.73	1	4425.68 Ca_7
4105.07	1	4105.16 Ce_3	4427.84	5	
4123.46	10		4430.26	8	
4137.21	1		4433.22	2	4433.28 Th_3
4141.95	6		4435.29	1	4435.30 Ca_8
4144.07	1	4144.13 Ce_3	4436.11	1	
4152.17	7	4152.20 Ce_8	4454.98	1	4455.07 Ca_9
4152.99	4		4456.07	2	4456.14 Ca_6
4192.47	5		4522.62	7	
4194.53	1		4525.50	3	
4196.69	6		4526.33	5	
4204.26	4		4554.38	2	4554.42 Ba_{10}
4215.85	2	4215.74 Sr_{10}	4558.72	4	
4217.85	6		4559.57	1	
4227.03	6	4226.89 Ca_{10}	4568.20	1	
4231.26	4		4570.33	1	
4238.67	8		4575.05	4	
4250.26	3		4580.37	2	
4263.76	6		4606.00	2	
4274.27	6		4613.54	4	
4275.82	4		4619.93	4	

The spectrum of lanthanum is characterized by a comparatively large number of *strong* lines.

SPECTRUM OF DIDYMIUM.

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4021.11	1		4262.08	1	
4021.53	1		4262.46	1	
4021.91	1		4266.92	1	
4023.11	1		4272.38	1	
4051.37	1		4272.91	1	
4056.74	1		4275.34	1	
4060.17	1		4280.98	1	
4061.28	3		4284.67	1	
4069.47	1	4069.39 Th_4	4298.01	1	
4075.41	1_n		4303.79	4	
4096.33	1		4304.65	1	
4096.96	1		4306.03	1	4306.04 Zr_3
4100.90	3		4314 66	1	
4109.20	2		4319.18	1	4319.15 Y_2
4109.61	4		4328.19	1	
4110.60	1		4329.21	1	4329.28 Y_3
4113.95	1		4338.94	1	
4116.93	1		4344.61	1	
4124.05	1	4123.95 Ce_7	4351.48	1	4351.51 Y_2
4133.46	1		4358.34	2	
4135.48	1	4135.55 Ce_4	4358.93	1	4358.93 Y_5
4143.08	1	4143.07 Ce_3	4368.53	1	Just
4143.33	2		4368.75	1	separated
4156.29	3		4375.12	4	4375.03 Y_{10}
4164.39	1		4385.85	2	
4165.20	1		4390.86	1_n	
4177.58	3	Double?	4398.17	1	4398.22 Y_6
4178.70	1		4409.02	1	4409.11 Ce_2
4179.56	2_n		4411.29	1	
4185.14	1		4412.47	1	
4289.64	1		4420.74	1	4420.71 Y_2
4211.57	1	4211.58 Zr_2	4421.39	1	4421.31 Y_2
4223.23	2		4422.83	1	4422.76 Y_5
4225.61	2		4425.51	1	
4232.60	1		4429.45	1	4429.44 Ce_4
4234.55	1	4234.44 Ce_2	4446.57	2	
4235.61	1		4450.09	1	
4236.28	1		4451.79	2_n	
4237.04	1		4456.59	1	4456.50 Zr_3
4240.22	1	4240.14 Ce_5	4463.13	1	4463.22 Y_2
4241.52	1	4241.41 Zr_5	4465.87	1	4465.73 Th_3
4247.65	2		4467.44	1	
4252.66	1		4502.00	1	
4256 65	1	4256.67 Zr_2			

THORIUM.

The carbon electrode was saturated with the nitrate of thorium, and a spectrum of a comparatively large number of lines was obtained, of which, without doubt, a number belong to other metals. The identification of the lines in the spectra of the different metals of the alkaline earths is especially difficult, since the latter cannot be sufficiently separated by chemical methods.

SPECTRUM OF THORIUM.

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4022.30	1	4022.40 Ce_3	4110.98	1	
4025.85	1		4112.43	1	
4026.42	1		4112.92	1	4112.84 Zr_2
4028.94	1		4113.71	1	
4032.74	1_n		4116.83	6	
4034.47	1		4122.02	1	
4036.70	2		4122.83	1	
4041.32	3		4123.70	1	
4043.21	1		4124.76	1	4124.88 Ce_4
4048.22	1		4131.63	2	
4048.61	1		4132.91	4	
4049.02	1_n		4134.21	1	
4069.39	4		4136.51	1	
4073.14	1		4140.35	2	
4077.89	2	4077.91 Sr_{10}	4141.75	2	
4082.03	1	4081.98 Zr_2	4142.63	2	4142.63 Ce_5
4082.43	1	4082.51 Zr_2	4142.80	2	
4085.21	5		4148.31	2_n	
4086.05	1	4085.91 Zr_4	4150.11	3	4150.15 Ce_9
4086.88	3	4086.97 La_{10}	4156.39	1	4156.49 Zr_5
4093.57	1		4156.67	3	
4094.90	4		4159.76	1	
4097.45	1		4162.88	2	Just
4097.85	1		4163.86	2	separated
4099.05	2		4164.45	1	
4100.50	1		4165.24	1	
4100.97	2	4100.90 Di_3	4168.81	2	
4103.35	1_n		4170.67	1	
4103.75	1		4171.00	1	
4104.47	2		4171.56	2	
4105.46	3		4178.16	5	
4106.03	1		4179.77	2	
4107.50	1	4107.55 Ce_4	4179.98	2	4179.93 Zr_7
4107.93	1		4181.18	1	
4108.55	5		4182.21	1	
4110.31	1		4183.57	1	
4110.70	1		4184.29	1	

SPECTRUM OF THORIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4184.87	1		4344.21	1	
4191.97	1	4192.01 Zr_2	4344.60	1	
4195.73	1	} Just	4347.46	1	
4195.97	1	} separated	4352.92	1	4352.99 Ce_4
4202.11	1		4353.64	1	4353.71 Ce_2
4206.93	1		4355.54	1	
4209.07	5		4361.52	1	
4211.72	1		4369.48	1	
4215.74	1	4215.74 Sr_{10}	4374.15	1	
4220.27	1		4374.98	2	4375.03 V_{10}
4226.98	4	4226.89 Ca_{10}	4377.50	1	
4229.73	1		4382.03	5	
4233.51	1		4387.98	1	
4240.84	1	4240.84 Ur_1	4391.29	5	
4244.17	1		4393.31	1	
4248.23	2		4395.14	1	4395.22 Zr_5
4249.91	1		4396.69	1	
4250.57	2		4398.16	1	
4256.34	1	4256.30 Ce_3	4399.39	1	4399.48 Ce_4
4263.54	1		4410.73	1	
4270.50	1		4412.90		} Not sufficient-
4271.26	1		4413.02	2	} ly divided
4273.51	4		4415.00	1	
4274.17	2	4274.27 La_6	4416.48	1	
4277.04	2		4418.97	1	4418.99 Ce_5
4277.47	4		4426.27	1	
4281.25	1		4427.95	1	
4281.61	1		4433.28	3	
4282.19	6		4435.25	1	4435.30 Ca_8
4283.67	2		4436.56	1	} Not sufficient-
4285.16	2_n		4436.82	1	} ly divided
4286.39	1		4439.42	2	
4288.20	1		4441.21	2	
4295.27	1		4448.14	2	
4298.79	1	4298.80 Zr_2	4455.19	1	
4302.74	1	4302.62 Ca_9	4461.43	1	4461.44 Zr_5
4306.58	1		4465.73	3	
4310.23	2		4474.37	1	
4318.61	1		4476.83	1	
4319.42	1		4483.23	1	
4319.82	1		4487.59	3	
4320.36	1		4510.78	3	
4320.84	1		4531.89	1	
4328.95	1		4532.54	1	
4329.78	1	4329.81 Zr_2	4533.55	1	4533.48 Zr_4
4332.18	1		4537.24	1	
4334.22	1		4554.31	1	
4335.97	2		4555.88	3	
4337.64	2		4563.55	1	
4341.30	1	4341.32 Zr_5	4589.37	1	
4342.50	1	4342.50 Zr_2	4619.60	1	
4343.93	1		4631.70	2	

YTTRIUM.

The spark spectrum of yttrium (chloride) has numerous lines in the violet, of which, however, as the following table shows, a considerable number may belong to other metals, especially lanthanum. From the spectrographic investigation of erbium (chloride) I obtained only such lines as also appear in the yttrium spectrum; it is, however, remarkable that the characteristic and strongest yttrium lines were absent. It may therefore be concluded that the erbium spectrum contains no yttrium lines, and that the lines common to the two spectra are to be removed from the yttrium spectrum. I have designated these lines which are without exception weak by Er ?; however, I do not venture to decide which of them are erbium lines, since no characteristic and strong erbium lines could be observed in the part of the spectrum investigated.

SPECTRUM OF YTTRIUM.

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4032.06	2	λ uncertain, 4031.88	4128.49	4	
4037.63	1	[La_6	4129.59	1	
4040.10	2		4130.57	1	
4041.06	2	Er ?	4134.06	1	
4043.21	2	4043.11 La_8	4137.48	1	
4050.08	1		4137.89	1	
4061.23	2	4061.28 Di_3	4143.01	4	
4073.94	2		4150.16	1	4150.15 Ce_9 [Er ?
4077.56	4	4077.58 La_9	4152.20	2	4152.22 Ce_8 2.17 La_7
4078.12	2		4156.27	2	4156.29 Di_3 Er ?
4083.92	2		4167.76	2	
4085.74	1		4174.41	2	
4086.93	2	4086.97 La_{10} Er ?	4177.81	10	
4098.78	2	Er ?	4179.73	1	4179.77 Th_2
4100.94	1	4100.90 Di_3 0.97 Th_2	4184.46	1	
4102.56	5		4186.87	2_n	
4103.50	2		4189.83	1_n	
4109.63	2	4109.61 Di_4 Er ?	4195.03	1	4195.00 Zr_5
4110.88	1	4110.88 Zr_2 Er ?	4196.63	1	4196.69 La_{10}
4111.57	1	4111.51 Ce_3 Er ?	4203.10	1	4203.18 Ce_5
4112.23	1		4204.84	4	
4114.03	1		4206.81	1	
4123.42	2	4123.46 La_{10} Er ?	4212.02	2	4212.17 Zr_7
4124.05	1	Er ?	4220.85	1	
4125.09	3		4223.14	1	4223.22 Di_2

SPECTRUM OF YTTRIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4225.60	2		4378.49	1	
4226.98	4	4226.89 Ca_{10} Er?	4382.27	1	4382.38 Ce_5
4229.93	1		4385.86	2	Er?
4232.62	1		4387.01	1	4387.04 Ce_5
4235.94	3		4387.86	1	4387.70 La_6
4236.92	1	4236.89 Zr_5	4391.08	2	Er?
4238.62	1	4238.67 La_8 Er?	4391.91	1	4391.93 Ce_5 Er?
4240.10	1	4240.14 Ce_5	4398.22	6	4398.22 Th_6
4251.98	1		4401.08	1	4401.09 Ce_2
4252.68	1		4403.46	1	
4253.75	1	4253.76 Zr Er?	4409.11	1	4409.11 Ce_2
4256.60	2		4409.63	2	
4262.25	1		4411.28	1	
4262.90	1		4418.99	1	4418.99 Ce_5
4263.74	1	4263.76 La_6	4419.79	1	
4272.00	1	4271.94 Ce_4	4420.71	2	4420.65 Zr_3 Er?
4279.84	2		4421.31	2	Er?
4280.93	2	Er?	4422.77	5	
4284.66	1		4424.50	2	Er?
4285.69	1		4429.49	1	Er?
4287.12	1	4287.12 La_7	4430.13	2	4430.26 La_8 Er?
4290.12	1	4290.16 Ce_6 Er?	4434.18	2	
4296.28	1	4296.25 La_6	4434.62	2	Er?
4296.94	1	4296.90 Ce_8 6.86 Zr_6	4434.99	1	
4302.67	2	4302.62 Ca_9 Er?	4443.94	1	4443.98 Ce_2 Zr_2
4306.20	2		4444.52	2	4444.48 Vd_4
4309.85	9		4449.65	1	4449.59 Ce_5
4314.74	1		4450.05	1	
4319.15	2	Er?	4450.99	1	4450.94 Ce_4
4327.37	1		4451.79	2	
4328.26	1		4452.93	2	
4329.28	2	Er?	4454.97	2	4455.07 Ca_9 Er?
4331.02	1		4458.75	2	Er?
4331.50	2		4460.45	2	4460.52 Ce_6 Er?
4334.35	2	4334.08 La_9 Er?	4463.22	2	Er?
4339.25	1		4465.68	2_n	Double? 4465.73 Th_3
4341.63	1		4467.19	2	
4342.45	2		4469.57	1	Er?
4346.77	1	4346.81 Zr_2	4471.26	1	
4348.08	2		4473.06	1_n	Double? 4473.04 Ce_3
4349.01	2		4477.19	1	
4351.51	2		4477.66	1	
4352.22	1		4478.89	1	
4358.93	5		4487.66	2	4487.59 Th_3
4361.02	1	4361.07 Zr_3	4506.12	2	
4362.33	1		4519.74	2	
4364.41	1		4522.49	1	4522.62 La_7
4364.96	1	4364.91 Ce_4	4523.18	1	
4366.38	1		4524.00	1	
4368.81	1		4527.34	2	4527.30 Ce_4 Er?
4375.03	10		4527.93	2	Er?

ZIRCONIUM.

This metal could be obtained in crystalline laminæ, which were, however, very small and fragile. A few pieces were found which it was possible to use as electrodes.

These laminæ quickly disintegrated in the induction spark, and in view of the rich and interesting spectrum I would gladly have used more compact pieces; such, however, were not to be had.

SPECTRUM OF ZIRCONIUM.

Wave-length tenth-meters	Intensity		Wave length tenth-meters	Intensity	
4018.56	4		4078.51	3	
4024.20	3		4081.38	6	
4024.66	6		4081.98	2	
4025.13	4		4082.51	2	
4027.42	4		4083.28	2	4083.38 Ce_8
4029.19	3		4084.48	3	
4029.85	7		4085.91	4	
4030.21	4		4087.88	2	
4031.58	2	4031.46 Ce_4	4090.68	6	Beautiful
4032.25	2		4090.97	3	double line
4034.34	3		4093.40	1	
4036.10	4		4094.48	2	
4040.46	3		4096.85	4	
4041.82	2		4099.52	2	
4042.45	3		4107.73	3	
4043.78	4		4108.59	3	4108.55 Th_8
4044.76	4		4110.19	3	
4045.92	6	4045.97 Ce_5	4110.85	2	
4048.78	8		4112.19	2	
4050.46	6		4112.84	2	
4054.54	2		4114.55	2	
4055.16	5		4119.38	2	
4055.84	4		4119.92	2	4119.94 Ce_7
4056.60	2		4120.32	2	
4057.94	2		4121.55	5	4121.52 Ce_6
4058.80	2		4128.08	2	4128.19 Vd_3
4060.32	2		4133.19	n	4133.95 Ce_8
4060.72	2		4135.88	3	
4061.66	3		4140.22	2	
4064.31	6		4146.12	n	
4068.93	2	4068.96 Ce_4	4147.94	2	
4071.26	3		4149.45	10	
4072.89	6		4151.18	8	
4075.14	4		4152.89	4	4152.99 La
4076.74	3		4156.49	5	
4077.25	3		4161.45	6	

SPECTRUM OF ZIRCONIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4163.98	n		4287.54	2	
4166.51	4		4289.33	2	
4169.49	2		4289.94	1	4289.82 Ce_7
4171.63	2	4171.75 Ur_8	4290.45	2	
4179.93	7		4291.20	2	
4183.43	3		4291.53	3	
4186.83	6	4186.70 Ce_8	4293.35	5	
4187.67	6		4294.98	5	
4191.77	2		4295.98	2	
4192.01	2		4296.86	6	4296.90 $_8$
4194.23	2		4298.80	2	
4195.00	5		4300.04	2	
4196.43	3	4196.54 Ce_5	4300.66	2	
4199.34	5		4301.21	2	
4201.71	5		4301.96	5	
4206.20	2		4302.99	5	
4209.23	10		4304.82	3	
4210.92	3		4306.04	3	
4211.58	2		4309.11	3	
4212.17	7		4309.96	2	4309.85 Y_4 / .93 Ce_4
4212.95	2		4312.38	2	
4214.15	5		4313.08	2	
4227.99	7		4315.10	5	
4231.88	6		4317.47	5	
4234.90	4		4319.21	2	
4236.37	5		4319.82	n	
4236.89	5		4321.28	2	
4237.72	3		4322.81	2	4322.74 La_8
4239.60	6		4324.17	2	
4240.57	5		4325.63	5	
4241.41	5		4329.81	2	
4241.99	6		4333.46	5	
4253.76	2		4336.55	n	4336.54 Ce_8
4254.55	2	4254.61 Cr_9	4337.91	4	4338.08 Ce_5
4256.67	2		4339.74	2	4339.73 Cr_4
4258.22	7		4341.32	5	
4261.47	2		4342.50	2	
4261.67	2		4343.39	2	
4263.36	2		4343.69	2	
4264.33	2		4345.32	n	
4265.13	2		4345.90	n	
4266.97	2		4346.81	2	
4268.19	5		4347.47	n	
4273.68	6		4348.22	6	
4274.92	3	4274.95 Cr_8	4359.01	2	4358.93 Y_8
4276.86	2		4360.01	9	4359.86 Cr_4
4277.50	2	4277.47 Th_4	4361.07	3	
4282.33	6		4366.63	4	
4285.46	n	4285.53 Ce_4	4371.17	7	
4286.20	2		4373.27	2	
4286.71	3		4379.98	8	

SPECTRUM OF ZIRCONIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4389.72	2		4491.75	2	
4390.68	2		4494.75	6	
4391.12	2	4391.29 Th_5	4495.69	2	
4395.22	5		4497.22	8	
4400.42	2		4501.47	2	
4401.54	2		4504.23	2	
4403.63	5		4507.33	4	
4413.24	3		4512.98	2	
4414.75	4		4518.26	2	
4420.65	3		4520.49	2	
4426.26	n		4523.00	2	
4427.49	3		4526.33	2	4526.33 La_5
4429.34	2	4429.44 Ce_4	4527.58	2	
4429.53	2		4531.35	2	
4431.74	3		4533.48	4	
4436.16	1		4534.19	2	
4437.09	1		4535.00	3	
4438.33	2		4535.98	6	
4440.72	5		4540.11	2	
4443.25	9		4542.37	4	
4443.98	2		4544.89	2	
4447.34	n		4548.98	2	
4449.31	2		4549.81	4	
4450.49	2		4552.58	1	
4451.12	2		4553.23	3	
4453.52	2		4554.24	5	
4453.89	5		4555.30	3	
4455.00	2	4455.07 Ca_9	4555.63	3	
4455.59	2		4558.23	2	
4456.50	2		4562.35	2	
4457.66	5		4564.04	2	
4460.57	2		4565.64	3	
4461.44	5		4572.24	2	
4467.13	2		4574.78	4	
4468.69	?		4575.73	4	
4469.70	2		4582.50	2	
4470.72	5		4590.66	2	
4480.95	3		4596.39	2_n	
4481.58	2		4602.70	4	
4482.56	3_n		4604.49	2	
4484.45	2_n		4614.07	4	
4485.76	2		4621.53	3_n	
4488.55	2		4626.42	3	
4489.38	2		4629.12	5	
4490.53	2		4633 91	4	

VANADIUM.

In this case a blue-green aqueous solution of vanadium chloride, with which the carbon electrodes were saturated, was used, since I could not obtain metallic venadium in suitable pieces. Hasselberg is at present engaged in an investigation of this metal, of which he received a piece fused in the electrical furnace of Moissan in Paris. He found that the mineral rutile, which he used in obtaining the spectrum of titanium, contains vanadium, and gives in a preliminary publication[1] the wave-lengths of a number of vanadium lines in the violet, which I have included in the following table:

SPECTRUM OF VANADIUM.

Wave-length tenth-meters	Intensity		Wave-length (Hasselberg) tenth-meters	Wave-length tenth-meters	Intensity		Wave-length (Hasselberg) tenth-meters
4023.64	1			4120.63	1		
4035.86	1	4035.91 Mn_6		4123.60	3		4123.68_3
4051.15	1			4128.19	3		4128.20_{3-4}
4051.53	1	4051.54 Ce_2		4132.16	4		
4057.28	1			4159.91	1		4159.79_{2-3}
4064.12	1			4174.20	1		
4071.68	1			4179.57	1		4179.56 Di_2
4077.86	3	4077.91 Sr_{10}		4181.05	1		
4090.69	2	4090.68 Zr_6	4090.73_3	4182.70	1		
4092.63	1	4092.54 Ce_5		4183.56	1		$4183.45_?$
4092.81	3	4092.87 W_4	4092.83_3	4190.00	1	4190.09 Mn_3	
4095.59	2		4095.65_3	4202.55	1		
4099.90	1		4099.94_{3-4}	4205.26	1		
4102.26	1			4210.07	1	4210.06 Cr_2	
4104.43	1	4104.47 Th_2		4215.77	3	4215.74 Sr_{10}	
4104.89	1	4104.80 Co_2		4225.52	1		
4105.27	4		4105.31_3	4226.98	7	4226.89 Ca_{10}	
4108.31	1			4232.74	2		
4109.85	4		4109.92_{3-4}	4233.20	1		
4110.04	6			4234.26	1		
4110.25	1			4234.79	1		
4112.39	1			4257.56	1		
4113.59	1			4259.55	1		
4115.26	4		4115.32_{3-4}	4262.36	1		
4116.57	3		4116.64_3	4268.85	2		4268.85_3
4118.31	1	4118.24 Ce_6		4271.76	2		4271.80_3
4119.53	1			4277.17	2		

[1] "Ueber das Vorkommen des Vanads in den skandinavischen Rutilarten." Stockholm, 1897. Also *Ap. J.*, **5**, 194.

SPECTRUM OF VANADIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length (Hasselberg) tenth-meters	Wave-length tenth-meters	Intensity		Wave-length (Hasselberg) tenth-meters
4283.17	1	4283.19 Ca_7		4420.14	1		
4284.24	2			4421.79	3		
4286.63	1			4422.47	1		
4288.02	1	4288.12 Ur_2		4423.41	1		
4289.57	1	4289.51 Ca_6		4424.15	1		
4292.05	2			4424.81	1		
4296.34	1			4425.73	1	4425.68 Ca_7	
4297.94	1			4426.26	3		
4298.30	1			4428.77	2		
4299.21	1	4299.16 Ca_5		4430.09	2		
4302.79	3	4302.62 Ca_9		4434.92	1		
4303.87	1	4303.77 Ce_2		4435.18	2	4335.30 Ca_8	
4306.44	1			4436.04	1	4335.99 Ca_7	
4307.44	1			4436.45	3	4336.57 Mn_5	
4308.06	1	4307.91 Ca_7		4438.14	4		
4310.03	1	4309.93 Ce_4		4441.96	4		
4318.91	1	4318.87 Ca_7		4443.61	1		
4330.32	3			4444.48	4		4444.41_{3-4}
4333.11	3		4330.15_3	4449.84	1		
4341.32	4	4341.32 Zr_5	4333.00_3	4451.28	1		
4343.14	1		4341.15_3	4452.29	4		
4353.14	5			4455.01	2	4455.07 Ca_9	
4355.30	1		4353.01_{3-4}	4456.11	2	4456.14 Ca_6	
4356.21	1			4456.82	1		
4363.79	1			4457.70	2	4457.73 Mn_5	
4364.45	1			4457.92	1		
4368.23	1			4460.02	5		
4368.80	1			4460.55	6		
4373.45	1	4373.44 Cr_2		4461.26	1	4461.24 Mn_5	
4374.04	1	4374.01 Ce_3		4462.62	3		
4375.50	1	4375.49 Cr_2		4465.79	1	4465.73 Th_3	
4379.39	8			4468.23	1		
4380.71	1		4379.40_{4-5}	4469.00	1		
4381.28	1			4469.92	3		
4384.89	7			4474.23	1		
4390.16	6		4384.85_{4-5}	4474.93	2		
4392.27	1		4390.11_{4-5}	4480.30	1		
4393.28	1			4489.06	2		
4394.06	1			4490.32	1		
4395.42	5			4490.95	1	4491.01 Ur_2	
4400.79	4		4395.40_4	4496.25	1		
4403.92	1			4496.93	1	4497.06 Cr_3	
4406.42	1			4502.11	1		
4406.86	6	Beautiful group of lines	4406.85_{4-5}	4514.23	1		
4407.83	6		4407.85_{4-5}	4524.28	1		
4408.44	5		4408.39_4	4529.64	1		
4408.72	5		4408.70_{4-5}	4530.93	1		
4412.34	1			4545.63	2		
4416.66	3		4416.70_3	4549.90	1	4589.81 Zr_4	

SPECTRUM OF VANADIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length (Hasselberg) tenth-meters	Wave-length tenth-meters	Intensity		Wave-length (Hasselberg) tenth-meters
4553.37	1			4580.63	3		
4554.29	3	4554.24 Zr_5		4586.62	3		
4560.99	1			4591.57	1	4591.57 Cr_2	
4572.06	1			4594.43	3		
4577.42	2			4606.48	1	4606.54 Ce_2	
4578.92	1			4619.87	1	4619.93 La_4	

URANIUM.

In the case of this very refractory metal I also made use of the chloride, which is soluble in water. Strong lines were not obtained in the violet part of the spectrum, but a considerable number of fine, sharp lines.

SPECTRUM OF URANIUM.

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4017.67	2		4117.75	1	4117.70 Ce_3
4019.11	1	4019.18 Ce_3	4124.93	2	4124.88 Ce_4
4026.16	2		4128.54	2	4128.49 Y_4
4033.59	1		4133.40	1	
4033.88	1	4033.80 Mn_2	4133.76	1	
4042.60	1		4135.98	1	
4042.91	2		4138.86	1	
4044.60	2		4139.34	1	
4050.24	2	4050.27 La_5	4141.44	2	
4052.11	2		4162.01	1	
4053.23	1		4162.74	1	
4054.51	1	4054.54 Zr_2	4163.87	2	4163.82 Cr_2
4058.38	2		4164.97	1	
4062.76	2		4165.82	2	4165.73 Ce_7
4067.97	2		4169.23	1	
4071.33	2	4071.26 Zr_3	4171.75	3	
4088.49	2		4173.12	2	
4090.08	3		4174.35	2	
4091.46	2		4189.37	2	
4097.97	2		4197.64	2	
4106.51	2		4204.54	1	
4107.10	1	4107.02 Ce_2	4206.61	1	
4116.27	2		4211.84	1	
4117.10	1	4117.10 Ce_3	4212.42	1	

SPECTRUM OF URANIUM—*continued.*

Wave-length tenth-meters	Intensity		Wave-length tenth-meters	Intensity	
4214.01	1		4362.40	2	
4214.59	1		4363.01	2	
4215.76	2	4215.74 Sr_{10}	4372.74	2	
4223.02	1		4373.59	2	
4226 96	4	4226.89 Ca_{10}	4393.81	2	
4228.96	2		4402.70	1	
4231.41	1		4406.07	1	
4231.91	1	4231.88 Zr_6	4406.72	1	
4232.24	1		4420.76	1_n	4420.71 Y_2
4234.85	1	4234.90 Zr_4	4423.28	1	
4240.84	1		4423.91	1	
4241.89	4	4241.99 Zr_6	4425.57	1	4225.68 Ca_7
4243.41	1_n		4426.87	1	
4244.58	3		4427.84	2	4427.84 La_5
4252.60	2		4434.11	2	
4267.44	2		4434.74	2	
4269.73	2		4435.84	1	
4274.06	1		4437.07	1	
4276.69	2		4450.83	1	
4278.35	1		4455.14	1	4455.07 Ca_9
4282.25	2		4463.06	2	
4283.28	1	4283.19 Ca_7	4465.38	2	
4288.12	2	4288.21 Ni_3	4472.61	3	
4289.09	1		4477.99	2_n	
4290.11	2	4290.16 Ce_6	4486.53	1	
4291.08	2		4491.01	2	
4295.60	1		4491.71	1	4491.75 Zr_2
4297.35	2		4493.28	1	
4301.91	2	4301.96 Zr_5	4506.33	1	
4302.71	2	4302.62 Ca_9	4510.51	2	
4310.55	1		4511.83	1	4511.86 Jn_{10}
4312.80	1		4515.48	2	
4313.34	1		4521.79	1	
4314.05	2		4538.42	2	
4323.95	1		4543.87	2	
4324.87	1		4545.77	2	
4327.20	1		4554.14	1	
4335.96	1		4555.28	1	4555.30 Zr_3
4336.70	1		4567.89	1	
4341.92	3		4570.14	1	
4347.37	2_n		4573.91	1	
4354.68	2_n	4354.64 La_5	4603.80	2	
4355.84	2		4605.31	2	
4357.92	1		4627.08	2	
4361.34	1		(4646.32)	2	λ uncertain

ON THE CONDITIONS OF MAXIMUM EFFICIENCY IN ASTROPHOTOGRAPHIC WORK.

PART I. GENERAL THEORY OF THE TELESCOPIC IMAGES OF DIFFERENT FORMS OF RADIATING SOURCES.

By F. L. O. Wadsworth.

The theoretical relations between aperture and focal length of an objective and the intensity of the images of celestial objects at its focal plane, have been discussed by a large number of different writers, among others Angot,[1] E. C. Pickering,[2] Grubb,[3] Harkness,[4] Searle,[5] Newall,[6] and W. H. Pickering.[7] All of the writers whose papers I have been able to examine have, with the exception of Angot and Newall, treated the subject from the standpoint of geometrical optics, and all of them without exception have failed to consider the influence of the aperture upon the intensity of the illumination at the focal plane due to the general (diffused) light of the sky. In regard to the method of treatment, I have already pointed out in previous articles[8] that geometrical considerations alone are almost certain to lead one to very erroneous conclusions (as has been the case with more than one writer of reputation in the case of the spectroscope), particularly when

[1] "Étude sur les Images Photographiques obtenues au foyer des Lunettes Astronomiques," *M. N.*, **37**, 387, May 1897.

[2] "An Investigation in Stellar Photography," *Mem. Amer. Acad. Sci.* **10**, 179, 1886.

[3] "On the Choice of Instruments for Celestial Photography," *M. N.*, **47**, 309, April 1887.

[4] "Astronomical Photography with Commercial Lenses," *Astronomy and Astrophysics*, **11**, 641, Oct. 1892.

[5] "Probable Advantages in Astronomical Photography of Short Focus Lenses," *Ibid.*, **12**, 575, Aug. 1893.

[6] "On the Formation of Photographic Star Disks," *M. N.*, **54**, 515, June 1894.

[7] "Investigations in Astronomical Photography," *Ann. Harvard College Obs'y*, **32**, Part I, 1895.

[8] See paper "General Conditions respecting the Design of Astronomical Spectroscopes," *Ap. J.*, **1**, 52, Jan. 1895; also paper "Further Notes on Astronomical Spectroscopes," *ibid.*, **3**, 176, March 1896; and "Conditions of Maximum Efficiency in the Use of the Spectrograph," *ibid.*, **3**, 321, May 1896.

dealing with sources of light of negligible angular magnitude, such as the stars, the smaller satellites and asteroids, and stellar nebulæ. In the present case the general failure to make use of the methods of physical rather than of geometrical optics is perhaps responsible for the oversight of the relations between intensity of general field and aperture, just alluded to.

The importance of this relation will be at once evident when it is remembered that the faintest image that can be either visually observed or photographically delineated on the sensitive plate must be somewhat brighter than the general field; and that it is, therefore, this *degree of contrast* between image and field rather than the *size* of the objective (except in so far as this latter influences the contrast) that determines the limiting magnitude of the faintest star or nebular detail that can ever be photographed.[1]

It becomes, therefore, of the greatest importance to define accurately the *relative* intensities of the images; I, of point sources (stars, asteroids, etc.), and II, of sources of finite though limited extent (nebulæ, comets, planets, etc.), in comparison with, III, extended luminous surfaces, such as the sky, which form the background against which, I and II, are seen or photographed. The expressions for all three cases are easily derived from fundamental considerations of the wave-theory, and have been for a long time well known. But in view of the considerable confusion of ideas that still seem to exist in regard to I and II (particularly I), and in view of the fact that the influence of III, seems to have been entirely lost sight of, it may be excusable to briefly outline the principal steps in the derivation of the expressions for the intensity of the physical images of point, line, and surface sources

1. The image of a mathematical point of unit brightness as seen from the image forming lens, and emitting monochromatic light of wave-length λ is a diffraction pattern whose center is coincident in position with the geometrical image, and whose intensity is represented by the general expression[2]

[1] "On the Conditions which determine the Limiting Time of Exposure for Photographic Plates in Astronomical Photography." *Knowledge*, Aug. and Sept. 1897.

[2] See Rayleigh, "Wave Theory," *Enc. Brit.*, **24**, §§ 11 and 12.

$$I_1^2 = \frac{1}{\lambda^2 f^2}\left[\int\int \sin\left(\frac{2\pi\xi}{\lambda f}x_1 + \frac{2\pi\eta}{\lambda f}y_1\right) dx_1\, dy_1\right]^2$$
$$+ \frac{1}{\lambda^2 f^2}\left[\int\int \cos\left(\frac{2\pi\xi}{\lambda f}x_1 + \frac{2\pi\eta}{\lambda f}y_1\right) dx_1\, dy_1\right]^2, \quad (1)$$

where f is the focal length of the image forming lens; ξ, η, the coördinates of a point in the diffraction pattern referred to its center (position of geometrical image); and x_1, y_1 the coördinates of a point in the diffracting aperture; the integration being extended over the whole of this aperture. If the latter is symmetrical with respect to x_1 and y_1, the first integral disappears ($\sin x$ being an uneven function), and the expression for I_1^2 reduces to the form

$$I_1^2 = \frac{1}{\lambda^2 f^2}\int\int \cos\frac{2\pi\xi}{\lambda f}x_1 \cos\frac{2\pi\eta}{\lambda f}y_1\, dx_1\, dy_1. \quad (1a)$$

Under these conditions the diffraction pattern is also symmetrical about the origin of coördinates, and if the aperture is circular, as is generally the case with a telescope, the intensity along the axis ξ is the same as along any other radial line r. Replacing ξ by r and integrating with respect to y along the ξ axis ($\eta = 0$), we at once obtain the second integral in the form

$$2\int_{-\frac{b}{2}}^{+\frac{b}{2}} \sqrt{\left(\frac{b}{2}\right)^2 - x_1^2}\, \cos\frac{2\pi r}{\lambda f}x_1\, dx_1$$

b being the diameter of the object glass.

By putting $x_1 = \frac{b\,w}{2}$ and $\frac{\pi b}{f\lambda}r = n$, we have for I_1^2

$$I_1^2 = \frac{1}{16}\frac{\pi^2 b^4}{\lambda^2 f^2}\left[\frac{4}{\pi}\int_0^1 \sqrt{1-w^2}\cos nw \,.\, dw\right]^2 = M \,.\, [\phi(n)]^2, \quad (2)$$

which is the form first obtained and computed by Airy.[1]

Similarly, by putting $x_1 = \frac{b}{2}\cos\phi$, we obtain

$$\left.\begin{aligned} I_1^2 &= \frac{1}{16}\frac{\pi^2 b^4}{\lambda^2 f^2}\left[\frac{2}{\pi}\int_0^\pi \cos\left(\frac{\pi b}{\lambda f}r\cos\phi\right)\right]\sin^2\phi \,.\, d\phi \\ &= M \,.\, \frac{4J_1^2(Br)}{(Br)^2} = M \,.\, 4\, z^{-2} J_1^2(z) \end{aligned}\right\} (3)$$

where $J_1(z)$ is Bessel's function of order unity.

[1] "On the Diffraction of an Object Glass with Circular Aperture," *Camb. Phil.*

This last (3) is the form due to Lommel,[1] who first applied Bessel's functions to the diffraction integrals.[2] These expressions represent, as is well known, the curve shown in Fig. 1 which

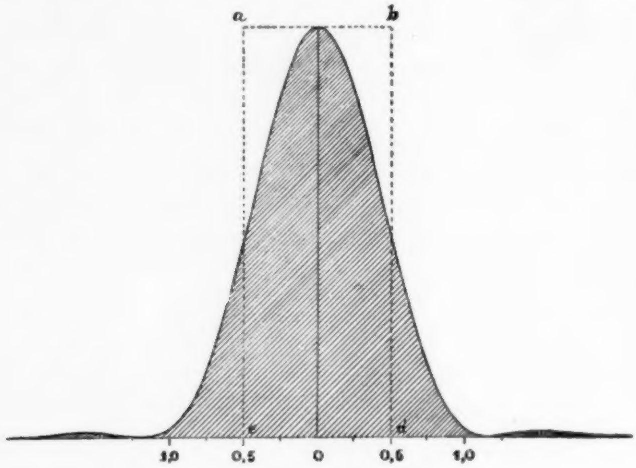


Fig 1

has been plotted from the values of $4\, z^{-2} J_1^2(z)$ given by Lommel.[3] The whole diffraction pattern is the symmetrical ring system

Trans., 1834; also "Undulatory Theory of Optics," 2d ed., 1877, pp. 78–80, and Table II, p. 159.

[1] *Lommel*, "Studien über die Besselschen Functionen," Leipzig, 1868; and *Z. f. Math. u. Phys.*, **15**, 166, 1870. See also Rayleigh, "Wave Theory," *Enc. Brit.*, **24**, § 12.

[2] My attention is called in this connection to an apparent blunder in one of my previous papers (*Ap. J.*, **1**, 57, line 10), due to the inadvertent omission of the words "*each point in*." The sentence in question should read "If the aperture is circular the diffraction image of each point in a vertical line (the bounding edge of the slit for example), etc. Strictly speaking, I *ought* to have made use of the expression for the diffraction image of a *line* instead of a point, in the case there considered. I did not do so, however, for the reason that for a circular aperture the former is much more complicated and less easily calculated (for plotting) than the latter, while the distribution in intensity (in a direction at right angles to the line) is so nearly the same in the two cases that the results arrived at (with reference to the effective broadening of a slit image by the effects of diffraction at the edges) would have been practically the same in the end. If the aperture is rectangular, as is generally the case in a spectroscope, they would be exactly the same.

[3] *Loc. cit.*

which would be obtained by revolving the figure about the axis of symmetry through the center o.

The lateral dimensions of the pattern vary as $\frac{\lambda f}{b}$, *i.e.*, the first minimum (dark ring) occurs at the point

$$r_1 = 1.22 \frac{\lambda f}{b} = 1.22 \frac{\lambda}{\beta}.$$

It is evident from either of these expressions that the intensity of illumination in the image of a point will depend on two things of an entirely different nature. *A*. On the brightness and character of the source of radiation. *B*. On the dimensions *i.e.*, aperture and focal length, of the image forming lens. It is not our purpose to deal with the first cause of variation at any length in this paper, which will concern itself chiefly with the effect of variations in the dimensions of the observing instrument under the different conditions met with in astrophotographic work. It may, however, be well for the sake of completeness, to briefly indicate the character of the various effects produced by variations of the first character, which as far as I have been able to find, have never before been considered, and indicate their bearing on some important problems in stellar photometry and stellar measurement, which will be considered more at length in subsequent papers.

A. Effect of variations in the value of λ.—If the light from the source of radiation were absolutely monochromatic, the intensity of the image would vary inversely as the square of the wave-length, a result due to the contraction of the linear dimensions of the pattern in the ratio of the first power of λ (see Fig. 1). But in general the radiation from any physical point, such as a star, is heterogeneous, the wave-length of the radiations ranging (theoretically at least) from 0 to ∞. The diffraction pattern resulting from the superposition of the individual images formed by the light of each wave-length λ, will be found by multiplying (2) or (3) by a factor, $\psi(\lambda)$ which expresses the distribution in intensity in the normal spectrum of a star as a function of the wave-length and integrating with respect to λ from 0 to ∞.

This gives us

$$I_R^2 = \int_0^\infty \psi(\lambda) I_1^2 \, d\lambda . \tag{4}$$

The expression I_R^2 represents the distribution in intensity in what may be termed the *real* diffraction image of a radiating point as formed at a focus of a perfectly achromatic telescope (for example, a perfectly figured reflector) of circular aperture and having the same coëfficient of transmission or reflection for radiations of all wave-lengths. In general neither the conditions of perfect achromatism not that of perfect equality of transmission will be fulfilled. In the first case the function I_1^2 would take the form[1]

$$I^2 = \frac{1}{e^2}[U_1^2 + U_2^2],$$

where

$$U_1 = \frac{eb}{fr} J_1(z) - \left(\frac{eb}{fr}\right)^3 J_3(z) + \ldots .$$

and

$$U_2 = \left(\frac{eb}{fr}\right)^2 J_2(z) - \left(\frac{eb}{fr}\right)^4 J_4(z) + \ldots ., \tag{5}$$

where e is the difference (measured towards the object glass), between the principal focal plane (at distance f from the object glass), and the focal plane for the radiation of wave-length λ. In the second case the function $\psi(\lambda)$ would simply have to be multiplied by a factor k_λ representing the coëfficient of transmission or reflection for each wave-length λ. Introducing this factor and reducing to the simplest form (r being constant as respects the integration for λ), we would have in the case of the achromatic objective from (3),

$$I_R^2 = \frac{b^2}{4r^2} \int_0^\infty k_\lambda \, . \, \psi(\lambda) \, . \, J_1^2\left(A\frac{r}{\lambda}\right) d\lambda . \tag{6}$$

The results obtained from the integration of (6) for different values of $\psi(\lambda)$ are of great interest in connection with the problem of measuring stellar temperatures. For since I_R^2 is a function of $\psi(\lambda)$, which represents the distribution of intensity in the

[1] LOMMEL, *Abhand. d. K. Bayer. Akad.*, 15, 235, 1886.

spectrum of the radiating point, and since this again is a function of the temperature of that point, it follows that a measurement of the distribution of intensity in the real diffraction image would afford us a means of measuring directly the temperature of the stars. At present we have no telescope large enough and no instruments sensitive enough to enable such measurements to be made ; but in view of the rapid progress in both of these directions during the last few years, it does not seem altogether improbable that before long such a method of investigation may become possible and practicable. The theoretical side of this problem will be discussed more at length in a paper which is now in course of preparation.[1]

A (*b*). In the case of visual observations what we are concerned with is not the distribution in intensity in the *real* diffraction image represented by (4) or (6), but the *effective* distribution in intensity as regards the eye. This will be obtained by integrating (4) only over that range of wave-lengths to which the observer's eye is sensitive. But since this sensitiveness is very different for different wave-lengths and is also relatively different for different observers, we must before integration introduce another factor, $L(\lambda)$, the ordinate to the "luminosity curve" for that particular observer. We then obtain

$$I_L^2 = \int_{\lambda_2}^{\lambda_1} k_\lambda \cdot \psi(\lambda) \cdot L(\lambda) \cdot I^2 \cdot d\lambda \tag{7}$$

which for the case of the achromatic objective becomes, as before,

$$\frac{b^2}{4r^2}\int_{\lambda_2}^{\lambda_1} k_\lambda \cdot L_\lambda \cdot \psi(\lambda) \cdot J_1^2\left(A\frac{r}{\lambda}\right) \cdot d\lambda \,. \tag{7a}$$

These integrals will be further considered in connection with the problem of determining the angular magnitudes of the stars from micrometrical measurements of the diameters of their diffraction rings, the elementary theory of which has already been developed by the writer.[2]

[1] "On a Method of Determining the Temperature of a Radiating Point from the Study of its Diffraction Pattern at the Focus of a Telescope," to be published in a future number of this JOURNAL.

[2] See paper "On the Resolving Power of Telescopes and Spectroscopes for Lines

A (*c*). In the case of photographic work we are similarly concerned with the *effective* distribution in intensity in the diffraction image with reference to the sensitive plate. As in the case of the eye, this will be obtained by introducing in (4) a factor P_λ (of the same nature as $L(\lambda)$), which represents the varying ordinate of the "actinic" curve for the particular kind of photographic plate which is being used, and integrating the resulting expression between the limiting wave-lengths λ_3 and λ_4, to which the plate is sensitive. This gives us

$$I_P^2 = \int_{\lambda_4}^{\lambda_3} k_\lambda \cdot P_\lambda \cdot I^2 \cdot d\lambda, \tag{8}$$

the value of I^2 being taken, as before, from either (3) or (5), according as the objective is or is not perfectly achromatic. This integral is of great importance in connection with the problem of the photographic determination of stellar magnitudes. At present these determinations are based on purely empirical formulæ, whose constants are determined by measurement of the photographic images of certain stars whose magnitudes have been determined by a different (*visual*) method. These formulæ are, therefore, only applicable to a (usually) very limited number of cases, and a more general formula derived from (8) and, therefore, based upon well-established theoretical considerations would, therefore, seem to possess certain very important advantages. It at least offers a logical explanation of the growth of the photographic image, which has been attributed, heretofore, to such manifestly inadequate causes as halation, photographic radiation, residual spherical aberration, etc. Further consideration of this subject is reserved for a future paper.

The integrals I_L^2 and I_P^2, (7) and (8), represent the distribution in intensity in what may be termed respectively (in contradistinction to the term used in connection with I_R^2) the "visu-

of Finite Width," *Mem. Spettr. Ital.*, **26**, 1, footnotes on pp. 6 and 7; *Phil. Mag.*, **43**, 317, footnotes on pp. 323 and 324. *Wied. Ann.*, **61**, 604, footnotes pp. 611 and 612. See also paper there referred to, "Application of Interference Methods to Astronomical Measurements," *Phil. Mag.*, **30**, 1, pp. 14 to 17. This theory will soon receive fuller consideration.

ally effective" and the "photographically effective" diffraction images.

Our knowledge of the individual functions k_λ, $\psi(\lambda)$, $L(\lambda)$, and $P(\lambda)$, which appear in (7) and (8), is unfortunately very limited. But for the purpose of integration we only need to know the products of the three functions

$$F_1(\lambda) = k_\lambda \cdot L(\lambda) \cdot \psi(\lambda), \tag{9}$$

and

$$F_2(\lambda) = k_\lambda \cdot P(\lambda) \cdot \psi(\lambda). \tag{10}$$

From the experiments of Abney,[1] Langley,[2] and others we can obtain $F_1(\lambda)$ and $F_2(\lambda)$ for various cases which correspond closely to those ordinarily met with in practice. These various cases will be considered more in detail in connection with the development of cases A, A (b), A (c), outlined above.[3]

B. Variations in the intensity of the image (I_1^2) *due to variations in the aperture and focal length af the observing instrument.*— Since not only the intensity but also the breadth of the diffraction pattern represented by (2) and (3) changes with the aperture b of the telescope, we must, in considering the relative brightness of the images given by two different instruments, fix upon some one point in the image as a basis of comparison, most conveniently its center (the position of the geometrical image). At this point $r = 0$, and

$$\phi(o) = 4\frac{J_1^2(o)}{o} = 1,$$

hence

$$i^2 = \frac{1}{16} \cdot \frac{\pi^2 b^4}{\lambda^2 f^2}. \tag{11}$$

Considering λ as constant (for its value take the mean wave-

[1] "On the Effect of the Spectrum on the Haloid Salts of Silver," *Proc. R. Soc.*, **33**, 164, 1881; "On the Comparative Effects of Different Parts of the Spectrum on Silver Salts," *Ibid.*, **40**, 251, 1886; "On Color Photometry," *Phil. Trans.*, **177**, 1886; "Transmission of Sunlight through the Earth's Atmosphere," *Ibid.*, **178**, 1887; etc., etc.

[2] "Energy and Vision," *Nat. Acad. Sci.*, **5**, 7, 1888.

[3] The writer hopes to have these results ready for informal presentation to the Astrophysical Conference to be held in connection with the dedication of the Yerkes Observatory, Oct. 18 to 22 (see p. 150 of this number).

length which is photographically or visually effective, which in the case of the refractor is or should be the wave-length of the light brought to minimum focus), we may put (11) in the form:

$$i^2 = \text{const } b^2 \beta^2, \tag{12}$$

where β is the angular aperture of the telescope objective. Hence we may say that for point sources the intensity at the center of the focal image varies directly as the product of the square of the linear aperture of the objective (or as its area), times the square of its angular aperture. The bearing of this result in photographic and visual work on the stars will be presently considered in connection with cases II and III.

II. *Intensity in the images of extended sources.*—The distribution in intensity in the image of a luminous source of finite size, whose elements vibrate independently, will be found by integration of the effects due to the individual points of which it is made up. Let x, y, be the coördinates of any point p in the source referred to any chosen point p_0, and ξ, η, the coördinates of a point in the focal plane referred to the geometrical image of p_0 as origin. Then the effects of the element at x,y, at ξ,η, will be

$$dI^2_{\text{II}} = M \cdot 4\,(Br)^{-2} \cdot J_1^2\,(Br)\; dx\; dy.$$

Let $\phi(xy)$ denote intensity at the point x,y, in the source. Then the total effect at the point ξ,η, due to the whole source will be

$$I^2_{\text{II}} = 4\,M \iint^{\text{Area}} \phi\,(xy)\,\frac{J_1^2\,(Br)}{(Br)^2}\;dx\,.\,dy, \tag{13}$$

where M and B have the values assigned in (3) and (6), and

$$r = \sqrt{\left(x\frac{f}{D} - \xi\right)^2 + \left(y\frac{f}{D} - \eta\right)^2}. \tag{14}$$

where D is the distance of the object from the image forming lens, and the quantities

$$x\,\frac{f}{D},\; y\,\frac{f}{D}$$

are, therefore, the focal plane coördinates of the geometrical image of p with reference to the origin $\xi = 0$, $\eta = 0$.

Instead of expressing the coördinates of a point in the focal plane in linear measure it is sometimes more convenient to

express them in angular measure. To do this we may put for small values of x, y, ξ, η:

$$\left.\begin{array}{ll} \xi = \gamma f, & \eta = \delta f, \\ x = \mu D, & y = \nu D. \end{array}\right\} \quad \text{Put also } \frac{\pi b}{\lambda} = \kappa.$$

Then

$$\left.\begin{array}{l} \dfrac{r}{f} = \theta = \sqrt{(\mu - \gamma)^2 + (\nu - \delta)^2}, \\ dx\, dy = D^2\, d\mu\, d\nu, \end{array}\right\} \tag{15}$$

and (13) becomes

$$I^2_{\text{II}} = MD^2 \int\!\!\int \phi(\mu\nu) \frac{4 J_1^2 (\kappa\theta)}{(\kappa\theta)^2}\, d\mu\, d\nu; \tag{16}$$

the integrations being extended over the whole of the source expressed in angular measure.

Still another expression which it is sometimes more convenient to use when the source is symmetrical is obtained directly from (1a) before integration. The effect at ξ, η, due to any element at x, y, is evidently[1]

$$\left.\begin{array}{l} I^2_{\text{II}} = \displaystyle\int\!\!\int\!\!\int\!\!\int \cos\frac{2\pi}{\lambda f}\left(\xi - x\frac{f}{D}\right) x_1 \cos\frac{2\pi}{\lambda f}\left(\eta - y\frac{f}{D}\right) v_1\, dx_1\, dy_1\, dx\, dy \\ = D^2 \displaystyle\int\!\!\int\!\!\int\!\!\int \cos\frac{2\kappa}{b}(\gamma - \mu)\, x_1 \cos\frac{2\kappa}{b}(\delta - \nu)\, y_1\, dx_1\, dy_1\, d\mu\, d\nu, \end{array}\right\} \tag{17}$$

the integration for $x_1 y_1$ being extended over the whole of the diffracting aperture as before, and that for x, y, or $(\mu, \nu,)$ over the whole of the source.

The integrations of the general expressions (13), (16), and (17) have only been accomplished for a few cases which, however, cover most of those met with in practice.

(1) *Luminous line of uniform intensity.*—In this case $\phi(xy) = 1$ and $x = dx$, Further, if the line is of any considerable length the intensity in the diffraction image at the point ξ will be the same for all values of η. Let us, therefore, consider the intensity along the line $\eta = 0$ We then get for r:

$$r^2 = \left(y\frac{f}{D}\right)^2 + \xi^2, \tag{18}$$

[1] MICHELSON "Visibility of Interference Fringes at the Focus of a Telescope," *Phil. Mag.*, **31**, 256, March 1891.

and for I^2_{11} from (13):

$$I^2_{11} = \frac{1}{16} \frac{\pi^2 b^4}{\lambda^2 f^2} dx \int_{-\infty}^{+\infty} \frac{4 J_1^2\left(\frac{\pi b}{\lambda f} r\right)}{\left(\frac{\pi b}{\lambda f} r\right)^2} dy. \tag{19}$$

If the line is infinitely long and infinitely narrow this evidently becomes in the limit equal to

$$2\, d\xi \int_0^{\infty} I^2\, d\eta. \tag{20}$$

In this form the integral has been investigated by Struve[1] and Rayleigh.[2] The latter finds for I^2_{11} the value

$$I^2_{11} d\xi = \tfrac{1}{8} b^2 \cdot \zeta^{-3} \cdot d\zeta \cdot K_1(2\zeta), \tag{21}$$

where

$$\zeta = \frac{\pi b}{\lambda f} \xi,$$

and[3]

$$K_1(2\zeta) = \frac{2}{\pi}\left\{ \frac{(2\zeta)^3}{1^2 \cdot 3} - \frac{(2\zeta)^5}{1^2 \cdot 3^2 \cdot 5} + \frac{(2\zeta)^7}{1^2 \cdot 3^2 \cdot 5^2 \cdot 7} - \ldots \right. \tag{22}$$

At the center ζ (or ξ) is zero, and we have (supposing λ constant as in (11)),

$$\left.\begin{aligned} i_{11} &= \frac{2}{3} \cdot \frac{b^3}{\lambda f} - \quad \ldots \\ &= \text{const}\ b^2 \beta \quad \ldots \end{aligned}\right\} \tag{23}$$

or the intensity at the center of the image of a long fine line varies as the square of the linear aperture times the angular aperture.

(2) If the line is short (although still long compared with its width), $dy = \frac{D}{f} d\eta$, and the intensity of the image is decreased in the inverse ratio of the focal length, the lateral distribution in intensity remaining the same. For such lines we have, therefore,

$$\left.\begin{aligned} i_{11} &= \frac{2}{3} \frac{b^3}{\lambda f^2} \cdot D \ldots \\ &= \frac{2}{3} \frac{b^3}{\lambda f^2} \quad \ldots \end{aligned}\right\} \tag{24}$$

[1] *Wied. Ann.*, **17**, 1008, 1882.
[2] "Wave Theory," *Enc. Brit.*, **24**, § 12.
[3] Rayleigh, "Theory of Sound," **2**, 151, § 302.

when the apparent brightness of the line is, as supposed heretofore, unity per unit of angular length. If λ is regarded as constant we have

$$i_{11} = \text{const}\ b\ \beta^2, \tag{25}$$

or the intensity at the center of the image of a short line will vary as the linear aperture times the square of the angular aperture.

(3) *Source of finite magnitude.*—The expression for the intensity in this case (given by (13), (16), or (17)) is in general very complicated. If the source is symmetrical, say circular, we can, however, obtain an expression for the intensity on the center, o, of the geometrical image with comparative ease. This case is most simply dealt with by using polar coördinates. At the center $\xi = o, \eta = o$, and the influence of all points in the source at a distance

$$r' = \sqrt{x^2 + y^2} = \frac{D}{f} r$$

from the center will evidently be

$$2\pi r' dr' = 2\pi \left(\frac{D}{f}\right)^2 r\, dr.$$

The total effect due to the whole source of radius R will be

$$i'_{11} = \frac{8\pi D^2}{f^2} M \int_0^{\frac{f}{D}R} \phi(r) \frac{J_1^2(Br)}{(Br)^2} r\, dr. \tag{26}$$

Put $Br = z$. Then $dr = \dfrac{\lambda f}{\pi b} dz$, and we have

$$i'_{11} = \frac{1}{2} \cdot \frac{\pi b^2}{f^2} D^2 \int_0^{z_1} \phi\left(\frac{z}{B}\right) z^{-1} \cdot J_1^2(z) \cdot dz. \tag{26a}$$

When the intensity is uniform over the source $\phi\left(\dfrac{z}{B}\right) = 1$, and the integral assumes the known form

$$\int_0^{z_1} z^{-1} \cdot J_1^2(z)\, dz = \tfrac{1}{2} [J_1^2(z_1) + J_0^2(z_1)], \tag{27}$$

and the value of i_{11} is, therefore,

$$\left.\begin{aligned} i'_{11} &= \tfrac{1}{4}\pi D^2 \cdot \frac{b^2}{f^2}[1 - J_1^2(z_1) - J_0^2(z_1)] \quad \ldots \\ &= \tfrac{1}{4}\pi\beta^2\left[1 - J_1^2\left(BR\frac{f}{D}\right) - J_0^2\left(BR\frac{f}{D}\right)\right] \ldots \end{aligned}\right\} \tag{28}$$

when the apparent brightness is unity per unit of angular surface as before.

When z_1 is small (equal, say, to dr) the value of $J_1^2(z_1) +$

$J_0^2(z_1)$ is simply $1 - \frac{z_1^2}{4}$. The total illumination at o is then

$$i'_{11} = M\pi(dr)^2, \tag{29}$$

as it evidently should be. When z_1 is large the sum of the squares of $J_1(z_1)$ and $J_0(z_1)$ tends toward zero, *i. e.*, the illumination at the center of the image of a large, uniformly illuminated area (which is the same as the illumination at any other point save near the edges of the image) becomes more and more nearly equal to

$$\tfrac{1}{4}\pi\frac{b^2}{f^2} = \tfrac{1}{4}\pi\beta^2, \tag{30}$$

as obtained on the principles of geometrical optics.

It is interesting to determine the size the object must attain before the influence of the term $J_1^2(z_1) + J_1^2(z_1)$ ceases to be felt. To show this I have calculated the following short table, which gives the value of this term for various values of z_1 expressed in terms of the resolving power of the telescope. To do this we put

$$\frac{f}{D}R = r_1 \text{ (the radius of the geometrical image),}$$

successively equal to $0.1\,af$, $0.2\,af$, $0.5\,af$, $1.0\,af$, $2\,af$. . . ., etc., where

$$af = m\frac{\lambda}{b}f$$

is the linear resolving power of the telescope objective of aperture b (for circular aperture $m = 1.1$).

TABLE I

Angular Diameter of Source	z_1	* $J_1^2(z)$	* $J_0^2(z)$	$J_1^2(z) + J_0^2(z)$
0.1a	0.3456	*0.0290	*0.9417	0.9707
0.2a	0.6911	0.1058	0.7816	0.8874
0.5a	1.728	0.3355	0.1458	0.4813
1.0a	3.456	0.0243	0.1396	0.1639
2.0a	6.911	0.0010	0.0891	0.0911
5.0a	17.28	0.0188−	0.0186+	0.0374
10.0a	34.56	†0.0092+	†0.0092−	0.0184
20.0a	69.11	†0.0046−	†0.0046+	0.0092

The intensity at the center of the images of small spherical sources (such as planetary nebulæ, asteroids, small satellites,

* Interpolated from Dr. Meissel's "Tafel der Bessel'schen Functionen," *Abh. d. K. Akad. d. W.* Berlin, 1888.

† Calculated from the semi-convergent series for $J_1(z)$ and $J_0(z)$.

etc.), may therefore be less than ten per cent. as great as required by the geometrical law of intensity, which is practically fulfilled *only* for points whose angular distance from the edge of the geometrical image is from ten to twenty times the resolving power of the telescope.

III. *Intensity of illumination of the field due to an indefinitely extended luminous area.*—In this case there is not strictly speaking any image, or rather the image is one whose edge lies at infinity. The general expression for the illumination at any point ξ, η, in the field will be given by (13), (16), or (17), the integration being extended from $-\infty$ to $+\infty$ for both x and y. In the problem with which we are most directly concerned (the effect of the illumination of the sky)[1] the intensity is approximately uniform over the whole illuminated area. In this case $\phi\,(xy) = 1$, and we have for the illumination at any point in the field (which is in this case evidently the same as for the point at the center):

$$I_{\text{III}}^2 = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} I_1^2\, dy. \tag{31}$$

which [as in the case of a luminous line, II, (2), equation (19)] is identical with

$$\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} I_1^2\, d\eta \quad \ldots . \tag{32}$$

This integral has been determined by Stokes,[2] who has shown that this integral is always equal to A, the area of the aperture, no matter what the form of the latter.

In this case $A = \dfrac{\pi b^2}{4}$, and the intensity at every point in the field of a telescope due to a uniform luminous area of infinite extent (more strictly an area extending over an entire hemisphere) is, therefore,

$$i_{\text{III}} = \text{const } b^2, \tag{33}$$

or the illumination of the field is proportional simply to the square of the linear aperture of the telescope, and is entirely independent of the focal length and angular aperture of the latter.

[1] See paper "The Effect of the General Illumination of the Sky on the Brightness of Field at the Focus of a Telescope." *M. N.*, **57**, 586, June 1897.

[2] *Trans. R. Soc. Edinburgh*, **20**, 317, 1853.

The expressions for i_{I}, i_{II}, i'_{II} and i_{III}, [(11), (23), (24), (28), (33)], as obtained above, enable us to express the contrast between image and field as functions of the aperture and focal length of the image forming lens, for most of the cases that arise in astrophotographic and astrospectrographic work. If i represents in general the intensity at the center of the image, and i_{III} that of the field, the contrast K will be represented by

$$K = \frac{i + i_{III}}{i_{III}} = 1 + \frac{i}{i_{III}}. \tag{34}$$

We have heretofore supposed that the intensity of the different sources considered has been such that their apparent brightness in each case has been unity. In actual practice, however, the apparent brightness will in general vary, and the intensity of the image will accordingly vary in the same proportion. If we let k be the factor expressing this variation the expression for the contrast becomes

$$K = 1 + \frac{k\,i}{k_{III}\,i_{III}}. \tag{35}$$

For the various cases which arise in practice we have:

(A) Contrast between point sources (stars and small asteroids, satellites, etc.), and general field due to III

$$K_A - 1 = \frac{k_a}{k_{III}} \frac{\pi b^2}{4\lambda^2 f^2} \qquad \text{from (11) and (13)}$$

$$= \text{const}\ \frac{k}{k_{III}}\ \beta^2. \tag{36}$$

(B) Contrast between line sources, and field,

(1) Long lines (meteor trails, lightning flashes, etc.)

$$K_B - 1 = \frac{k_b}{k_{III}} \cdot \frac{8}{3} \cdot \frac{b}{\pi\lambda f} \qquad \text{from (23) and (24)}$$

$$= \text{const}\ \frac{k_b}{k_{III}}\,\beta. \tag{37}$$

(2) Short lines (star trails, short meteor trails, bright line stellar spectra formed by an objective spectroscope and lengthened by motion parallel to slit, etc.),

$$K'_B - 1 = \frac{k'_b}{k_{III}} \cdot \frac{8}{3} \cdot \frac{b}{\pi\lambda f^2} \qquad \text{from (24) and (33)}$$

$$= \text{const}\ \frac{k'_b}{k_{III}} \cdot \frac{1}{f} \cdot \beta. \tag{38}$$

(C) Contrast between finite sources of nearly uniform intensity over a considerable area, and field (nebulæ, comets, Moon, Sun, etc.),

$$K_C - 1 = \frac{k_c}{k_{III}} \cdot \frac{1}{f^2} \quad \text{from (28) and (33)} \tag{39}$$

(D) Contrast between line sources and finite sources of uniform intensity (linear markings on lunar and planetary surfaces, etc.).

In such cases the lines are always short and we have from (24) and (28).

$$K_D - 1 = \frac{8}{3}\frac{k_b'}{k_c} \cdot \frac{b}{\pi\lambda} \cong \frac{k_b'}{k_c} \cdot \frac{1}{a}\,, \tag{40}$$

a being the angular resolving power of the telescope.

The conclusions which follow from a consideration of the cases A and C (equations (36) and (39)), so far as the problems met with in astrophotographic work are concerned, have been presented in another paper.[1] Case B has been considered in its application to the problem of photographing meteor trails and meteoric spectra.[2] Case D has also been considered in connection with the problem of planetary observations, both visual[3] and photographic.[4] The consideration of cases A, B, and C in their applications to spectrographic and other astrophysical investigations will be taken up in the October number of this JOURNAL.

YERKES OBSERVATORY,
June 1897.

[1] "On the Conditions which Determine the Limiting Time of Exposure of Photographic Plates in Astronomical Photography," *A. N.*, Aug. 1897, and ***Knowledge*, 20,** 193, Aug. and Sept. 1897; see also note "The Effect of the General Illumination of the Sky on the Brightness of Field at the Focus of a Telescope," *M. N.*, **57**, 193, June 1897.

[2] "On the Most Efficient Forms of Instrument for the Photographic Observation of Meteors and on Some New Methods of Determining their Parallax and Absolute Velocity."— To be soon published, probably in the *Astronomical Journal.*

[3] "On the Effect of the Size of an Objective on the Visibility of Linear Markings on the Planets."—To be published in the September number of the *Observatory.*

[4] "On the Photography of Planetary Surfaces; with Introductory Note on the Applications of Photography to Astronomical Research;" to be published in the October number of the *Observatory.* See also paper "On a Comparison of the Photographic and of the Hand and Eye Methods of Delineating the Form and Surface Markings of Celestial Objects," *Pop. Astron.*, **5**, 200, August 1897.

Minor Contributions and Notes.

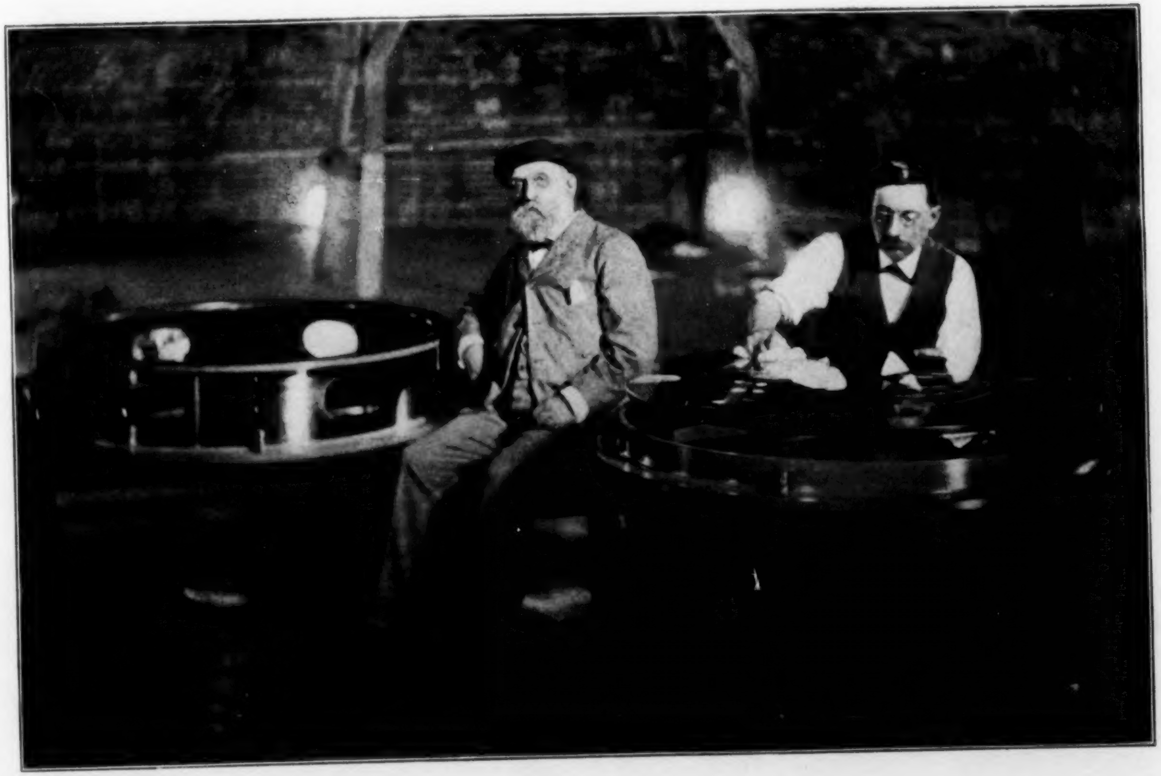
ALVAN GRAHAM CLARK.

It was with great regret that the scientific world learned of the death of Mr. Alvan G. Clark, which occurred at his residence in Cambridge, June 9, 1897. The immediate cause of his decease was apoplexy, although he had been in ill-health for some time previously. In anticipation of a more extended biography, the following sketch of his life has been prepared by one who stood near him, at the request of the family.

Mr. Clark was born in Fall River, Mass., July 10, 1832. His father, Alvan Clark, was born in Ashfield, Mass., March 8, 1804; he was a descendant of Thomas Clark, one of the early Pilgrim settlers. His mother was Maria (Pease) Clark. He had two sisters, Caroline and Maria Louise, and one brother, George Bassett, who was born in Lowell, February 14, 1827, and died in Cambridge, January 2, 1892.

While a student at Andover, George attempted the construction of a small reflecting telescope. In this way the attention of the father was first directed to optical pursuits, and about 1850 the firm of Alvan Clark & Sons was founded. The superiority of their lenses soon attracted the attention of astronomers, especially that of the Rev. W. R. Dawes, who introduced several of their larger productions abroad. Their instruments gradually increased in size, until in 1861 all former attainments were surpassed by the construction of one of 18¾ inches aperture for the Northwestern University, now located at Evanston, Ill. While testing this glass at Cambridge, Mr. Clark discovered the companion to Sirius, for which he was awarded the Lalande prize of the French Academy of Sciences. The Princeton University refractor, of 23 inches aperture, marked the next increase in size, after which two lenses of 26 inches diameter were made, one for the United States Naval Observatory at Washington, the other for the Leander McCormick Observatory of the University of Virginia. Then came the 30-inch refractor for the Imperial Observatory at Pulkowa, for which a gold medal was awarded by the Russian government. Finally, in

PLATE XVI



ALVAN G. CLARK AND CARL LUNDIN WITH THE CROWN LENS OF THE FORTY-INCH OBJECTIVE.

YERKES OBSERVATORY, MAY 21, 1897.

1887, the famous Lick telescope was constructed, which ended the joint productions of the Clarks.

During the last five years Mr. Clark executed the 20-inch lens for the Denver Observatory; one of 24 inches aperture for Mr. Percival Lowell; the 24-inch photographic objective for the Harvard Observatory station at Arequipa, Peru; and finally, as a crowning triumph, the great Yerkes lens, 40 inches in diameter. This last he accompanied to its destination and superintended its final mounting only a few days before his death. In addition to his optical work, he was a member of several governmental eclipse expeditions, and, like his father, the discoverer of a number of close double stars.

In his domestic life he was exceedingly fortunate. He married, January 2, 1865, Mary Willard, daughter of Joseph A. Willard, who rendered him constant devotion throughout their married life. She died July 10, 1892. They had one son, Alvan, who died in youth, and three daughters.

In personal appearance and social intercourse Mr. Clark was unusually attractive. With finely cut features, of sympathetic nature, and serene temper, he drew to himself a host of friends. Fond of the companionship of intelligent men, interested in all that pertained to science, and delighting in travel and reminiscence, he had a storehouse of facts from which to draw. These he invested with a peculiar charm as he shared them not only with personal friends but the welcome visitor. An hour at his workshop was an unusual pleasure, both for the scientific man as well as the ordinary individual, and no one was ever turned away.

His sympathies were broad and deep, and his regard for his family and friends exceptionally strong. His love for the best literature was intense. Shakespeare and the poets were his especial favorites, and, endowed with a remarkably retentive memory, he could quote from them almost indefinitely.

To him death was the inevitable sequel of life, the gate to be opened by a kind and all wise Providence, and so without fear he met the future.

Thus has passed away the last of a trio of remarkable men, who had scaled the heighths of perfect achievement, and but a few days ago gave to the world its greatest masterpiece. O. C. Wendell.

Mr. Clark's sudden death is especially felt by those who have been brought into close touch with him through his invaluable ser-

vices to the Yerkes Observatory. It was no small proof of devotion to his work and interest in its successful termination that he should be willing to leave his home after a nearly fatal stroke of apoplexy and to undertake a journey of over a thousand miles in order to accompany the forty-inch objective to its destination. We may well believe that he experienced no small degree of satisfaction in the safe arrival of the objective, its successful installation, its admirable performance, and its fortunate escape from injury in the subsequent accident to the rising-floor. The members of the staff of the Yerkes Observatory, fully recognizing the extent of their indebtedness to Alvan Graham Clark, unite with all friends of science in mourning his loss.

With characteristic hope of further progress, Mr. Clark was considering at the time of his death the possibility of constructing an objective still larger than his last great masterpiece. While he feared the effect of flexure, he felt that it might perhaps be possible to still further increase the aperture without endangering the performance of the objective.

It is a pleasure to add that the well-known firm name of Alvan Clark & Sons will still continue to be used. The men who shared with Mr. Clark the work of building great objectives and mountings will continue the business under the direction of Mr. Carl Lundin, who served the firm for twenty-five years. It gives the writer great pleasure to certify, after some acquaintance with his work, to Mr. Lundin's great ability as an optician. GEORGE E. HALE.

EDWARD JAMES STONE.

THE death of the Radcliffe Observer, so long and so favorably known for his work in England and at the Cape, comes as a serious loss to British astronomy. His star catalogues are indispensable to every astronomical library, and every observer has had reason to be grateful for his accurate places of both northern and southern stars.

Born in London in 1831, he went to Cambridge at a rather mature age, and carried off several honors. In 1860 he was appointed Chief Assistant at Greenwich, where he remained seven years, chiefly engaged with meridian observations, under the directorship of Airy. In addition to his observational work, he published various important memoirs, including determinations of the constants of nutation and refraction, investigations of the proper motion of stars, the motions of the solar

system in space, etc. He also received, in 1869, the Gold Medal of the Royal Astronomical Society, for his work at Greenwich in rediscussing the observations of the Transit of Venus in 1769.

Following his seven years with Airy came his mission to the Cape, where in a period of ten years he produced the well-known Cape Catalogue of 12,441 stars. Then came his appointment as Radcliffe Observer, a post which he held until his sudden death on May 9. Here he completed the Radcliffe Catalogue of 6424 stars, and conducted a variety of important investigations in other fields of work. In spite of his advanced years, he observed the last total eclipse in Nova Zembla, and was planning to visit India for the same purpose in 1898. Had he lived, it is certain that he would have made many more contributions to the literature of astronomy, as he retained to the last all his exceptional vigor of mind and body.

ARMINIO NOBILE.

We have received from the Royal Observatory of Naples an announcement of the death of Arminio Nobile, Second Astronomer of the Observatory, and Professor of Geodesy in the University of Naples. Professor Nobile was best known for his papers on geodetical subjects, which include many important contributions on determinations of latitude and longitude and related investigations. In addition to this he carried on much astronomical work, principally on double and multiple stars. He was a member of the Accademia dei Lincei and other important scientific societies, to whose publications he was a frequent contributor.

ADAM HILGER.

In these days of refined physical and astronomical measurements, and, as an eminent physicist has put it, of "discoveries in the sixth place of decimals," we are sometimes inclined, in our award of honors to those who have been directly concerned with some great advance in science by reason of some brilliant individual discovery or generalization, to forget, for the time being, the due proportion of praise due to those who have played a less conspicuous, though not less important, part in the history of science; but who, by the skillful and intelligent performance of the part allotted to them, whether it be, on the one hand the improvement of the refined instruments of research of mod-

ern observational science, or on the other the painful accumulation of accurate data by the faithful daily performance of routine and therefore at time irksome tasks; have been the ones who have in reality made these discoveries and generalizations possible. It is only when two such men as Hilger and the last of the famous firm of Alvan Clark & Sons are suddenly taken from us within a few weeks of each other, that we are brought to reflect upon, and realize fully, how large and important a part a few famous instrument makers and opticians of the latter part of this century have played in the development of modern observational astronomy, particularly that branch of it, the recent birth and growth of which have recently been so ably described by its founder, Sir William Huggins.[1] "By the death of Hilger . . . . the physical sciences, and especially astronomical physics, have suffered a loss which cannot immediately be made good. Standing in the front rank of practical opticians, he did much to promote scientific progress along various lines, and his thoroughly scientific training enabled him to undertake work of the highest character."[2]

Adam Hilger was born in Darmstadt in 1839, and for some years after attaining his majority he followed the profession of mechanical engineering in his native city. He then entered the famous establishment of Ertel in Munich. From there he went first to London, then to Paris, where he was engaged for some years with the firm of Lerebours & Secretan, and constructed many scientific instruments under the immediate direction of Foucault. Early in the seventies he returned to London, and after five years' work with Browning he set up the establishment of his own at Islington which has since become so famous. His work was held in the highest repute, both by scientific men and by his fellow artists. It is said that there is not a laboratory or observatory of importance in all England that does not possess one or more of his instruments and there are a great many in this country. A large part of the initial equipment of the Allegheny Astrophysical Observatory came from Mr. Hilger's workshops.

Of Mr. Hilger's personal characteristics there are many who can speak much better than the writer, who met him only once, in the fall of 1892, but who has always remembered that occasion with great

[1] "The New Astronomy: A Personal Retrospect," *Nineteenth Century*, June 1897.

[2] From a biographical notice by Mr. Fowler in the June number of the *Observatory*, to which the writer wishes to acknowledge his indebtedness for the few facts in regard to Mr. Hilger's life and scientific career which appear in this article.

pleasure. But his earnestness and enthusiasm in his work; his perfect readiness to show and explain the details of processes, which might by many other men in his position have been zealously guarded as trade secrets; his kindliness and simplicity of manner, and the entire absence of any element of self-consciousness or personal vanity when talking of his own work, combined to produce a personality that made a deep impression upon even those who knew him but slightly.

F. L. O. W.

ON THE ACTION OF COHERERS.

THAT a tube of filings changes its resistance when electric radiation falls on it is in itself an interesting fact. Questions arise. Do the filings *move* so as to make better contact? Does the process of electric welding take place as has been suggested by Lodge? Or is the medium changed by the radiation so that conduction of the current takes place? Experiments to throw light on these questions and also to determine the sensitiveness and constancy of various receivers and the influence of the dielectric were carried on during the autumn quarter of 1896 in the Ryerson Physical Laboratory.

(1) Tubes of glass containing iron filings, magnesium powder or graphite in air gave readings differing from each other by as much as 30 or 40 per cent. The mirror did not always return to zero upon tapping. The effect was ten times as strong when the radiation took place in oil as when it occurred in air. The response to long waves was feeble.

(2) Paraffin substituted for air made the receiver more sensitive but harder to regulate. Gelatine, on the other hand, allowed a current to flow and no change was observed after the radiation.

(3) A receiver with about one dozen steel spheres in a glass tube which they just fitted was very sensitive. Even when the end of a wire from the induction coil was brought within a distance of half a meter the mirror of the galvanometer (an ordinary D'Arsonval which, in this and the succeeding experiments, was short circuited by one-third ohm resistance) was thrown through twenty or thirty degrees. When the wheels of a Wimshurst machine three or four meters away, were turned through a small angle, though no sparking had yet taken place, a large deflection of the needle resulted. Bringing up a charged Leyden jar to the receiver did not affect it, but discharging the jar even by a moist

thread caused a large deflection. The effect therefore is due to sudden alteration in the strength of the field and is not similar to that of a secondary wire on a light suspended system—which is due to electrostatic action.

(4) A tube containing iron tacks 1^{cm} long in heavy paraffin oil or in lubricating oil was not nearly so sensitive as that of (3) but was far more constant. It was not affected by the turning of the Wimshurst machine until a spark passed between its knobs. This receiver, as well as that of (3), came accurately back to zero after tapping.

(5) Mercury, well shaken up in oil, formed into little globules varying from one to one-tenth millimeter in diameter. This, when placed in a glass tube, responded to the radiation, but was not very constant and was hard to tap back.[1]

(6) The receivers used have responded feebly to long, and very strongly to short, waves. An attempt was made to obtain a receiver having a definite period of vibration. Such a receiver would, if its vibrations were not too strongly damped, respond almost entirely to a definite radiation. For this purpose two small brass rods 6^{cm} long and about 4^{mm} diameter, were mounted end to end on hard rubber, one rod being fixed, the other capable of slow motion along its length. Though we used a delicate spring to produce this motion, the adjustment could not be made until a few drops of the oiled mercury of (5) were introduced between the adjacent ends. A small rubber tube over these ends served to keep the mercury in its place. Iron particles might have been used in place of the mercury. This receiver responded strongly to short waves and hardly at all to the long waves, but it was very inconstant compared with that of (4).

While writing this paper I noticed in *Nature*, June 17, 1897, an abstract of Mr. W. H. Preece's paper in which he described the receiver used by Mr. Marconi in signaling without wires. It is the same as the receiver just described. It did not appear as sensitive as the receiver of (3) or as that of (7).

(7) To make the adjustment more delicate two brass spheres two and one-half centimeters in diameter, were suspended by two fine cop-

[1] MR. ROLLO APPLEYARD, in the *Phil. Mag.*, May 1897, describes some interesting experiments on the action of strong electric fields on oiled mercury. He finds that the mercury globules run into one another. On the other hand, though I used the best microscopes obtainable here, no motion of the particle due to radiation could be observed.

per wires 15^{cm} long, to one of which was fastened a horizontal spiral spring. By a motion of the spring, the pressure between the spheres could be regulated. This proved to be the most sensitive receiver used, but it was affected by air currents and jars outside and inside the building.

(8) To remove the effect of air currents, to test the influence of heat and light on the receiver and its action in a vacuum the spheres were suspended in a jar which could be exhausted. No difference could be noticed between the action of the receiver in vacuum and in air except that in the former condition it was steadier. Throwing heat rays on the receiver did not decrease perceptibly its resistance, but, on the other hand, when the current had been set up by the electric radiation the heat rays always tended to bring the mirror back to zero.

(9) To see if any motion of the spheres occurred on account of the radiation, there was attached to one of the spheres a small mirror which was made one of the mirrors of an interferometer. Though great precautions were taken against air currents and vibrations, these effects could not be gotten rid of. The experiment, though unsatisfactory on this account, indicated that there was no motion of the spheres, and further, that if electric welding occurred it was of the most delicate nature.

(10) Finally, to test the influence of the dielectric, various oils, such as vaseline oil, mixtures of vaseline and lubricating oil, Venice turpentine, etc., were used in the receiver of (4). This general law was found — the more viscous the medium, the less lasting is the change of resistance and at the same time the less liable are the nails to be disturbed by the tapping back process. A medium which will allow the change of resistance to be permanent and yet sufficiently viscous not to allow the nails to be disturbed by the tapping back process, and such that after tapping the galvanometer returns to zero, is the one required for a quantitative receiver. A mixture of vaseline and lubricating oil was found to serve the purpose fairly well.

(11) *Conclusion.* The receiver is made more sensitive and at the same time less constant by decreasing the number and increasing the size of the conducting parts of the receiver.

The change of resistance may be accounted for thus. The electric spark between the conductors brushes aside the thin layer of non-conducting medium, and makes the intervening medium conducting. Whether this change will prove permanent or not depends on the

viscous and capillary forces of the surrounding medium. For air these forces are small so that the change should be permanent, as it usually is.

I am much indebted to Professors Michelson and Stratton for suggestions in connection with these experiments.

G. F. HULL.

RYERSON PHYSICAL LABORATORY,
THE UNIVERSITY OF CHICAGO,
June 1897.

ON THE MODE OF PRINTING MAPS OF SPECTRA AND TABLES OF WAVE-LENGTHS.

As a number of objections have been made to the mode of printing maps and tables adopted by the ASTROPHYSICAL JOURNAL, and as the question as to the most suitable mode has been reopened for discussion, it seems desirable that the arguments for and against the different systems should be stated, essentially as they were brought up for consideration at the first meeting of the Editorial Board, when the method now in use was adopted. In preparing the following statement I have consulted the minutes of the meeting referred to above.

1. *Mode of printing maps.*—If a map of the spectrum is printed with the red end toward the right, the letters assigned to the principal lines by Fraunhofer read in the reverse order. This seems to be a matter of very small importance. On the other hand, if the red end is placed on the left, the wave-lengths run in the reverse order. So far as convenience in reading the position of a line is concerned, this also seems to be a consideration of small importance, since a scale is read from right to left just about as easily as from left to right. A much greater weight attaches to the following consideration: it is frequently desirable to represent quantities of various kinds graphically, as functions of the wave-length, and observance of the usual convention of analytic geometry places the red end of the spectrum on the right. It was pointed out by Professor Rowland that concave grating spectroscopes are always constructed so that the wave-lengths increase towards the right, in accordance with the same convention.

The existence of unexplored regions at the ends of the spectrum does not affect the question of printing maps, as additions can be made equally well at either end.

The question of precedent was regarded as one of importance. In the following table is shown the usage of a number of prominent investigators, particularly of such as have published maps which are frequently consulted by spectroscopists. I have made a considerable number of additions to the names brought up at the meeting, and the list could be made much longer. With each authority is given the name of one map, or work in which a map is contained.

RED END TOWARD THE RIGHT.

Kirchhoff.—"Untersuchungen über das Sonnenspectrum."

Ångström.—"Recherches sur le spectre solaire."

Cornu.—"Sur le spectre normal du Soleil; partie ultra-violette."

Rutherfurd.—Photographic Map of the Diffraction Solar Spectrum.

Draper.—Photographic Map of the Diffraction Solar Spectrum.

Rowland.—Photographic Map of the Normal Solar Spectrum.

Higgs.—Photographic Atlas of the Normal Solar Spectrum.

Thalén.—"Mémoire sur la determination des longueurs d'onde des raies métalliques."

Huggins.—"On the Spectra of Some of the Chemical Elements."

Liveing and Dewar.—"On the Ultra-Violet Spectra of the Elements."

Vogel.—"Untersuchungen über das Sonnenspectrum."

Lockyer.—"Researches in Spectrum Analysis."

Abney.—"The Solar Spectrum from λ 7150 to λ 10000."

Langley.—"Researches on Solar Heat."

Pickering.—"The Draper Catalogue."

Young.—"The Sun."

RED END TOWARD THE LEFT.

Fraunhofer.—"Bestimmung des Brechungs- und Farbenzerstreuungs-Vermögens verschiedener Glasorten."

Lecoq de Boisbaudran.—"Spectres lumineux."

Fierez.—"Étude du spectre solaire."

Piazzi Smyth.—"Micrometical Measures of Gaseous Spectra."

Hartley and Adeney.—"Measurements of the Wave-lengths of Lines of High Refrangibility in the Spectra of Elementary Substances."

Thollon.—"Spectre solaire."

L. Becker.—"The Solar Spectrum at Medium and Low Altitudes."

Hasselberg.—"Untersuchungen über die Spectra der Metalle."

Kayser and Runge.—"Ueber die Spectren der Elemente."

Eder and Valenta.—Researches on the spectra of the elements.

The mode chosen (doubtless at random) by Fraunhofer was reversed by Ångström, who has been followed by many other spectroscopists. It will be seen that while a good many maps which chiefly serve the purpose of illustration have the red end toward the left, most of the maps of the solar spectrum used as standards of reference (notably the maps of Ångström and Rowland) have the red end toward the right.

The analogy with the keyboard of a piano, which has been urged as an argument in favor of placing the red end of the spectrum on the left, was not, I think, brought forward at the meeting of the Editorial Board. To me its importance seems to be merely that of a convenient mnemonic.

2. *Mode of printing tables.*—There seems to be no preponderance of authority in favor of either mode of printing wave-length tables. The Potsdam tables and the tables of Rowland begin with the short wave-lengths. The tables of Thalén, Kayser and Runge, and Hasselberg begin with the long wave-lengths, as do the tables in Watts' "Index of Spectra." Some authors have used both methods.

Tables of wave-lengths will have to be added to at both ends as a result of future investigations. The infra-red region of the spectrum is the more extensive; on the other hand, the ultra-violet region contains the greater number of lines.

A strong argument for beginning tables with the long wave-lengths has arisen in the last few years through the discovery of extensive line series in the spectra of the elements. The natural numbers, substituted successively in the formulæ of Rydberg and Kayser and Runge, give a table of lines which begins with the long wave-lengths. It is possible, however, that future modifications of these formulæ may make it convenient to reverse this order. There are also series which run in the opposite direction from that above mentioned. The decision of the board that tables should begin with the short wave-lengths was therefore to a large extent arbitrary.

My personal opinion is that the practice of printing maps with the red end toward the right should not be changed, but that there is no objection to reversing the present mode of printing tables.

James E. Keeler.

THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO. BULLETIN NO. 2.

COMPLETION OF THE YERKES TELESCOPE.

AFTER many months of labor in erecting the mounting of the Yerkes telescope, Messrs. Warner & Swasey had early in May so far completed their work as to permit the 40-inch objective to be attached to the tube. The objective had been stored in the workshop of Messrs. Alvan Clark & Sons since October 1895, the date of its acceptance by Mr. Yerkes.[1] It therefore remained to transport it from Cambridgeport, Massachusetts, to Williams Bay, Wisconsin. Through the courtesy of the officials of the Wagner Palace Car Company and the Boston & Albany, New York Central, Lake Shore & Michigan Southern, and Chicago & Northwestern Railway Companies, a private car was furnished for the express purpose of carrying the objective, and free transportation was given over the respective roads. Mr. Marvin Hughitt, President of the Chicago & Northwestern Railway, to whom the Yerkes Observatory was already indebted for many signal favors, provided a special locomotive to bring the car containing the objective from Chicago to Williams Bay.

The flint and crown lenses were sewed up in soft cloth and packed in curled hair in separate boxes. They arrived safely at Williams Bay on May 19 in the charge of Mr. Alvan G. Clark and Mr. Lundin, who had come for the purpose of putting together the objective and attaching it to the telescope. This work was successfully completed on the following day, but on account of cloudy weather no observations could be made until the evening of May 21. On this and every other clear night up to May 29 the telescope was used with very satisfactory results. The seeing, which had been very fine during the warm days of early May, was not nearly so good in the exceptionally cold and unseasonable weather which prevailed during the latter part of the month. Nevertheless many objects were well seen, notably the Ring Nebula in Lyra, the great cluster in Hercules, and the Dumb-bell Nebula. The great light-gathering power of the telescope was well illustrated by the fact that Professor Barnard saw these and other objects better than he had ever seen them at Mt. Hamilton. An observation made by him, though of no special astronomical impor-

[1] For an account of the testing of the objective by Professor Keeler and the writer, see the ASTROPHYSICAL JOURNAL, 3, 154, February 1896.

tance, further testifies to the excellence of the 40-inch objective. Near Winnecke's companion to Vega, in a region which Professor Burnham had frequently examined with the Lick telescope, Professor Barnard saw and measured a faint star (Pos. 312°; Dist. 53″). Its distance from Vega is far too great to permit us to suppose that the two objects are physically connected; but the detection of a hitherto unseen star near so thoroughly observed an object as Vega affords some indication of the light-gathering power and perfection of polish of the 40-inch objective. Professor Burnham observed with the telescope on one night. Although the conditions were not favorable for a thorough test, his long experience enabled him to form a judgment regarding the optical performance of the instrument, with which he expressed himself as very much pleased. Taken together with the tests of the objective made by Professor Keeler and the writer in 1895, these favorable indications lead us to expect that the last great work of Alvan G. Clark will do honor to his memory.

In most respects the mounting of the large telescope proved to be very satisfactory. Even before there had been any opportunity to make the usual adjustments or to rate the driving-clock, a star brought to the center of the field of an eyepiece giving a power of 1000 would remain for a long time without apparent drift. With the telescope unclamped in right ascension a star under this power of course appeared to move very rapidly across the field of view. But as soon as the electric clamp was applied the star seemed to stop instantly, without vibration or drift, although the clock was called upon to set in motion a mass weighing some twenty tons. The steadiness of driving was admirable, the star images appearing almost motionless with reference to the micrometer wire.

The quick-motion motors were found to move the telescope well, and proved to be very useful for purposes of reversing and for rough settings. They could not be used to produce such small motions of the tube in both right ascension and declination as are required in picking up a star, on account of the long trains of gearing and shafting between the motors (in the clock room at the head of the column) and the moving parts. It was a matter of considerable difficulty to move the telescope by hand, but Messrs. Warner & Swasey, who had not completed their work when these observations were made, expect to be able to reduce the friction very decidedly at certain points, and to make the instrument as easily manageable as the Lick telescope.

The usual adjustments were being made when a most unfortunate accident deprived us of the use of the telescope.

ACCIDENT TO THE RISING-FLOOR.

On May 29, at 6:43 A. M., the south side of the rising floor fell to the ground from a height of 45 feet. The north side was forced against the vertical steel guides, and descended only a short distance. Several of the iron treads were stripped from the spiral stairway on the telescope column, but though it must have been badly jarred the instrument seems to have been otherwise uninjured. It is very fortunate that so little damage was done. No one was in the building when the accident occurred; Professor Barnard and Mr. Ellerman had been observing during the greater part of the night, but had gone home at dawn. Shortly before leaving they had raised the floor to within six inches of its highest level, where it was left for the convenience of Messrs. Warner & Swasey's men, who intended to continue in the morning their work of adjusting the mounting. An examination of the wreck at once revealed the cause of the accident. The wire cables which supported the floor had not been properly attached to it, and one or more of them had pulled out of the fastenings.

Messrs. Warner & Swasey, whose contract included the telescope mounting, dome, and rising-floor, undertook the work of reconstruction with little delay. New steel was shipped to the Observatory to take the place of the injured members, and in a few weeks the old floor had been taken down and another erected in its place. The work was pushed rapidly forward, and by the middle of August the rising-floor was once more ready for use. Great pains have been taken to make the new cable fastenings secure, and a repetition of the accident is certainly not to be feared.

APPROXIMATE POSITION OF THE YERKES OBSERVATORY.

The following provisional coördinates of the Yerkes Observatory have been determined by Mr. W. H. Wright, Fellow in Astronomy, with a 1¼-inch Bamberg universal instrument. The construction of the instrument does not permit of its use as a zenith telescope in the ordinary sense of the term. The vertical circle is read by micrometers, however, and with a slight modification of the usual procedure, Talcott's method has been employed. The longitude, which is not regarded as anything more than a rough approximation to the truth, has been

determined from comparisons of a sidereal chronometer with the daily time signal of the Western Union Telegraph Company at the Williams Bay station.

Approximate latitude of the Yerkes Observatory $= +42° \ 34' \ 15'' \pm$
Approximate longitude of the Yerkes Observatory $= 5^h \ 54^m \ 14^s \pm$ W.

DEDICATION OF THE YERKES OBSERVATORY.

The formal dedication of the Observatory will take place on October 21-22, 1897.[1] In connection with the dedication a series of informal conferences on astronomical and astrophysical subjects will be held at the Observatory on October 18, 19, 20, and 21. Although certain details remain to be arranged, the following provisional programme may perhaps be of interest at the present time:

Provisional Programme.

Oct. 18, Monday.

2:30 P.M. Fourth annual meeting of the Board of Editors of the ASTROPHYSICAL JOURNAL.

4:30 P.M. Opening session of informal conferences.
Informal talks on recent investigations, including:
Professor Wadsworth on the Application of Diffraction Phenomena to Astronomical and Astrophysical Measurements.
Dr. Hull on Electric Radiation.
(Other titles may be added.)

7:30 P.M. Professor Wadsworth will demonstrate with the 40-inch Yerkes telescope the application of interference methods to astronomical measurements.

Oct. 19, Tuesday.

9:00 A.M. Second session of conferences.
Professor Crew on the Source of the Characteristic Spectrum of the Metallic Arc.
Professor Hale on a Remarkable Change in the Reversing Layer near a Sun-spot.
Dr. Humphreys on the Effect of Pressure on Wave-length.
Professor Keeler on the Spectra of Stars of Secchi's Third Type.
Professor Lord on Researches in Stellar Spectrography, (the spectrograph of the Emerson McMillin Observatory will be exhibited).
Professor Runge on Oxygen in the Sun.

[1] Necessarily postponed from October 1.

Oct. 19, Tuesday—*continued.*

Dr. Wilczynski on Hydrodynamical Investigations of the Solar Rotation.

Professor Stone on the Great Nebula of Orion.

(Other titles may be added.)

2 : 15 P.M. Address on the Yerkes Observatory by Professor George E. Hale, Director.

3 : 00 P.M. Professor Hale will show various solar phenomena with the 40-inch Yerkes telescope, including the chromosphere and prominences, the reversal of the H and K lines in prominences and faculæ, the duplication of the D_3 line, etc.

Mr. Ellerman will exhibit the solar spectrum, including the infra-red and the ultra-violet regions (heliostat room.)

Experimental demonstrations will be given in the Observatory laboratories as follows:

The effect of pressure on wave-length (Dr. Humphreys).

Measurements of wave-lengths in the infra-red spectrum (Professor Wadsworth).

Analysis of electric radiation by means of the interferometer (Dr. Hull).

Experiments with the rotating arc and the "hooded" arc (Professor Crew).

Demonstrations in the Optical Shop:

Process of grinding a 5-foot speculum (Mr. Ritchey).

Exhibition of Foucault's method of testing the figure of mirrors (Mr. Ritchey).

Exhibition of plane parallel plates and other optical surfaces. Demonstrations of methods of testing. (Dr. Brashear).

Demonstrations in the Instrument Shop:

The instrument shop will be in operation, and a 24-inch heliostat will be shown in process of construction (Mr. Lorenz).

Wadsworth's method of making a perfect straight-edge (Mr. Mors).

Rowland's method of grinding a perfect screw (Mr. Mors).

7 : 30 P.M. Professor Barnard will show the following objects with the 40-inch Yerkes telescope:

N. G. C. 224 (Andromeda Nebula).

N. G. C. 598.

N. G. C. 1976 (Orion Nebula).

N. G. C. 2245 (cometary nebula).

N. G. C. 2392 (planetary nebula).

Oct. 19, Tuesday—*continued.*

N. G. C. 6543 (planetary nebula).
N. G. C. 6618 (Swan nebula).
N. G. C. 6720 (annular nebula).
N. G. C. 7009 ("Saturn" nebula).
N. G. C. 7078 (globular cluster).
R. Leporis (Hind's crimson star).
Selected variable stars.

The 12-inch refractor and Mr. Ritchey's 24-inch reflector will be used for miscellaneous observations.

Oct. 20, Wednesday.

10:30 A.M. Third session of conferences.

Professor Comstock on Determinations of Stellar Parallax and on Investigations of the Lunar Atmosphere.
Professor Doolittle on the Latitude Work of the Flower Observatory.
Professor Rees on the Variation of Latitude and the Reduction of the Rutherford Photographs.
Professor Myers on the System of β Lyrae.
Professor Pritchett on Personal Equation in Longitude Determination.

(Other titles may be added).

2:30 P.M. Fourth session of conferences.

Professor Barnard on Astronomical Photography (illustrated with lantern views).
Professor Hough on Jovian Phenomena.
Professor Pickering on the Work of the Harvard College Observatory.
Father Hagen on an Atlas of Variable Stars.
Professor Poor on a New Form of Mirror for Reflecting Telescopes.
Father Hedrick on the Photochronograph (the instrument used at the Georgetown College Observatory will be shown).

7:30 P.M. Professor Hale will show the spectra of the following objects with the 40-inch Yerkes telescope:

N. G. C. 1976 (Orion nebula).
N. G. C. 7027.
α Tauri.
α Orionis.
α Cygni.
α Ursae Majoris.
α Cassiopeiae.
α Canis Majoris.

Oct. 21, Thursday.

9:30 A.M. Final session of conferences.

Dr. Laves on the Teaching of Theoretical Astronomy in America and on Jacobi's Investigations in Theoretical Astronomy.

(Other titles may be added.)

11:00 A.M. Arrival at the Observatory of the Trustees, members of the Faculty, students, and guests of the University of Chicago.

11:30 A.M. Formal presentation and acceptance of the Yerkes Observatory.

1:00 P.M. Luncheon served to offiçlal guests, Trustees and members of the Faculty.

2:00 P.M. to 3:30 P.M. Inspection of the Observatory.

4:00 P.M. Departure for Chicago of the special train provided for the Trustees and official guests.

8:30 P.M. Reception to Mr. and Mrs. Yerkes, the visiting men of science, and members of the Observatory staff.

Oct. 22, Friday.

10:00 A.M. Inspection of the Ryerson Physical Laboratory and other buildings of the University of Chicago.

In the Ryerson Laboratory Professors Michelson and Stratton will demonstrate the effect of a magnetic field on radiation, and exhibit an interferential comparer and a new form of harmonic analyzer.

1:00 P.M. Luncheon given by the President of the University to the visiting men of science and other official guests.

3:00 P.M. Address by Professor Simon Newcomb, LL.D.

7:00 P.M. Banquet to the visiting men of science.

GEORGE E. HALE.

YERKES OBSERVATORY,
July 1897.

A NOTE ON THE EFFECT OF HEAT ON PHOSPHORESCENCE.

IN connection with note on the above subject in *Nature*[1] of June 3d attention should be called to the recent and important work in the same field of Professors Wiedemann and Schmidt, who within the

[1] "Effect of a Change of Temperature on Phosphorescent Substances," Mr. Ralph Cusack, *Nature*, **56**, 102, June 3, 1897.

last two years have published in *Wiedemann's Annalen* and elsewhere[1] several very interesting papers dealing with the luminescent properties of various substances in both the solid and the fluid state. This work will be reviewed more at length in a subsequent number of this JOURNAL. In the present note I will only refer to certain experiments on the effect of a change in temperature on the phosphorescent properties of solids, which are described in their paper "Über Luminescenz von festen Körper und festen Lösungen," published about a year and a half ago.[2] The results of these experiments are summed up in the following conclusions:

7. Ein vorheriges Erhitzen wirkt auf die luminescirenden Substanzen in doppelter Weise: *a*) durch das Erhitzen werden die Substanzen (z. B. Strontiumsulfat) im allgemeinen dichter, bez. in andere Modificationen übergeführt oder, *b*) chemisch verändert.

   In beiden Fällen kann die Farbe des Leuchtens durch das Erhitzen wesentlich verändert werden.

8. Je stärker ein Körper bei seiner Darstellung erhitzt worden ist, desto länger leuchtet er nach. Diese Regel gilt ausnahmslos.

10. Für den Einfluss der Temperatur des luminescirenden Körpers auf das Leuchten ergiebt sich:
    a) die durch die starken Kathodenstrahlen hervorgerufene Luminescenz bleibt von -80° bis zu den höchsten erreichten Temperaturen ca. 500° erhalten;
    b) bei niederen Temperaturen ist die Intensität der Luminescenz[3] grösser als bei höheren;
    c) das Nachleuchten verschwindet bei höheren Temperaturen; es ist bei -80° länger als bei 0°;
    d) die Farbe ändert sich manchmal so, dass zu den bei den niederen Temperaturen vorhandenen Strahlen bei höheren Temperaturen brechbarere hinzutreten, vgl. $M_gSO_4 + 1\ \%$ M_nSO_4, $Q_nSO_4 + 1\ \%$ M_nSO_4. (Versuche bei niederen Temperaturen.)

Der Einfluss der Temperatur auf die Wellenlängen der Chemieluminescenz wäre also ähnlich demjenigen auf die Wellenlängen eines

[1] See this JOURNAL, **3**, 207, March 1897.

[2] *Wied. Ann.*, **56**, 112, October 1895.

[3] Bei Erregung mit den schwächeren Sonnenstrahlen liegen die Verhältnisse etwas anders.

in gewöhnlicher Weise glühenden Körpers; auch bei letzterem verschiebt sich das Maximum der Emission mit steigender Temperatur nach dem Violet.

The effect of intense *preliminary* heating in increasing the phosphorescent power of barium sulphide was independently observed by the writer while engaged in experiments on the preparation of phosphorographic plates for solar work in September and October 1895, just before the paper of Professors Wiedemann and Schmidt appeared. A note in reference to this observation was published in this JOURNAL for November 1896.[1] At that time, owing to temporary lack of library facilities, I had not seen the paper in *Wiedemann's Annalen*, or I should certainly have referred to it. I take this opportunity of doing so and of according to the authors full priority both of observation and of publication. F. L. O. WADSWORTH.

YERKES OBSERVATORY,
July 22, 1897.

ON THE REVERSING STRATUM AND ITS SPECTRUM, AND ON THE SPECTRUM OF THE CORONA.[2]

THE observation made by the writer in 1870, described on pages 82 and 83 of the last edition of *The Sun*, received a beautiful photographic confirmation during the total eclipse of 1896. Mr. Shackleton, the photographer of an English party at a station in Nova Zembla (the only party which was not baffled by bad weather), secured an instantaneous photograph at the critical moment with a so-called "prismatic camera," which is simply a camera with (in this case) two large prisms in front of its lens, no collimator being used — a photographic "slitless spectroscope."

When the Sun's disk is reduced to an extremely narrow crescent by the encroaching Moon, this crescent itself acts like the slit of an ordinary spectroscope, and photographs taken with such an instrument immediately before totality are just like the usual solar spectrum, except

[1] "Note on the Preparation of Phosphorescent Barium Sulphide," *Ap. J.*, 4, 308, November 1896.

[2] The above note will appear as an addendum in a forthcoming edition of Professor Young's well-known work, *The Sun*. The editor of the ASTROPHYSICAL JOURNAL has in his possession copies of Mr. Shackleton's remarkable photographs, which will be reproduced as soon as Sir Norman Lockyer desires to have them published.

that the dark Fraunhofer lines are replaced by dark crescents—*negative* images, so to speak, of the still uncovered portion of the disk. As soon, however, as the photosphere disappears, the remaining, much fainter crescent is simply the solar atmosphere, and if the observation of 1870 is correct, its photograph ought to show a series of *bright* images replacing the former dark ones, and it did.

Mr. Shackleton watched the waning crescent with a small direct-vision prism held in the hand, and at the instant when the brilliant dark-line spectrum vanished he "pressed the button" and caught on his plate the "flash-spectrum," as it has been called by Mr. Lockyer. The exposure was about half a second. The photograph shows a long range of several hundred bright, curved images, of which there are nearly 250 in the blue portion of the spectrum between F and H. About twenty-five are much more extensive and conspicuous than the others, and are images of the chromosphere and prominences. They are due to hydrogen, calcium, helium, strontium, and one or two other elements which often appear in the chromosphere. The rest are simply reversals of the Fraunhofer lines, as Mr. Shackleton has shown by developing the flash-spectrum into a bright-line spectrum of the usual form (which is easily done by a simple mechanical contrivance), and comparing it with an ordinary dark-line solar spectrum photographed with the same camera and prisms, but with the addition of a collimator and slit. The agreement is practically complete, although there are two or three somewhat conspicuous Fraunhofer lines which are missing in the flash-spectrum, probably because they originate not above the surface of the photosphere, but in its depths, as probably also do the wide hazy shadings that accompany the H and K lines and some others, but this is a matter for further investigation.

A second photograph, taken not more than five or six seconds later, shows only the chromospheric images, proving of course that the stratum of the solar atmosphere which produces the Fraunhofer lines by its absorption must be extremely thin. This is perfectly in accordance with the view expressed on pages 325 and 339, and does not at all favor the opposite "Dissociation Theory" of Mr. Lockyer, according to which the lines, many of them at least, are produced only at a considerable elevation, where the temperature is low enough to allow the recombination of elements dissociated in the hotter regions underneath.

A photograph made by the same instrument about the middle of

the eclipse, with an exposure of nearly a minute, shows very finely the green coronal ring, corresponding to the old "1474 line," and several others in addition. These are all in the violet part of the spectrum, and are extremely faint, excepting one which is a little below H. They are all probably due to the same hypothetical element, still unidentified, but provisionally named "coronium." The photograph also seems to make it certain that ***hydrogen, helium,*** and ***calcium,*** though brilliantly conspicuous upon the plate in the images of the prominences, ***are entirely absent from the corona,*** a result agreeing with that deduced from similar photographs made in 1893, but only recently published. It is quite clear that the earlier observations (referred to on pages 260, 261, and 262) were misleading from the fact that the apparatus did not sufficiently guard against the effects of illumination of the air by light from the prominences.

C. A. YOUNG.

NOTE ON THE PRESENCE OF VANADIUM IN RUTILE.

IN connection with Professor Hasselberg's "Note on the Chemical Composition of the Mineral Rutile" in the last number of this JOURNAL, Professor Rowland wishes to have it stated that he discovered all the important vanadium lines in the spectrum of rutile some four or five years ago. He also found traces of vanadium in specimens of titanic acid, and noticed that the strongest of the vanadium lines were given in Kayser and Runge's tables as iron lines.

NOTE ON THE RELATIVE FREQUENCY OF THE H AND K LINES IN THE SPECTRUM OF THE CHROMOSPHERE.

IN Sir William and Lady Huggins' interesting and important article (p.77) reference is made to the fact that H and K are recorded with relative frequencies of 75 and 50, respectively, in Young's *Catalogue of the Chromosphere Lines.* It has seemed to me desirable to point out that in all probability a relative frequency of 100 would have been ascribed to both lines had photographic rather than visual methods been employed in Professor Young's very important work at Mt. Sherman. During my four years of solar work at the Kenwood Observatory I do not remember that I ever photographed the ultra-violet spectrum of

the chromosphere and prominences without recording both of these lines. Moreover, K is almost invariably stronger than H in such spectra. The only way in which I can account for the values of the relative frequency given by Professor Young is by supposing that his eye is decidedly more sensitive to H light than to the more refrangible light of the K line. I think Professor Young will agree with me as to the contradictory evidence afforded by the photographic method.

I mention this point, not because it has any bearing upon Sir William Huggins' valuable conclusions, but rather because it would seem that in this critical region of the spectrum, photographic results are to be preferred to those obtained visually.[1]

GEORGE E. HALE.

NOTICE REGARDING REPRINTS.

THE attention of contributors to the ASTROPHYSICAL JOURNAL is called to the fact that hereafter *one hundred* reprints, bound in covers, of each article accepted for publication will be furnished to the author free of charge, provided a request to this effect is sent with the manuscript.

[1] Since the above note was written I have received the following letter from Professor Young, which goes to confirm the opinion expressed regarding the relative frequency of the H and K lines: "The numbers given in my catalogue of chromosphere lines were intended to represent the relative frequency with which I was able *to observe them* in 1872; and K is a good deal more difficult to observe *visually* than H, from being nearer to the limit of ordinary visual observation in the spectrum. Later, by the help of the fluorescent eyepiece I carried the limit up above 3875, and was able to observe H at 3889. Even before your photographic operations I had become satisfied that both H and K were *always* present in the chromosphere spectrum, though I was not able to observe them both. I have not my books with me, and cannot now give references, but am very sure that I had printed that opinion more than ten years ago, probably in one of my 'spectroscopic notes' in the *American Journal of Science*. I think it quite likely, as you suggest, that my eye falls off more rapidly in sensitiveness towards the violet end of the spectrum than is the case with many. I know that Dr. Brackett can always see further above K than I can: in fact, with me it is usually pretty hard to see K at all except with the interposition of a purple glass to cut off the rest of the light."

REVIEWS.

Die Gravitations Constante, die Masse und mittlere Dichte der Erde, von DR. CARL BRAUN, S. J. Abhandlung der Mathematisch Naturwissenschaftlichen Classe der Kaiserlichen Akademie der Wissenschaften, Wien, Band 44, pp. 74+iii.

A LITTLE over a year ago (April 1896) the writer reviewed in this JOURNAL the important work of Professor Boys on the determination of the Newtonian constant, which was then considered to have given us a value "fully ten times better than any preceding determination . . . . and . . . . for the first time comparable (in accuracy) with the results attained in our other physical measurements." The work of Dr. Braun in this same field, which is fully described in the above memoir, is perhaps less elegant and finished than that of Professor Boys as regards some of the details of the design, construction, and manipulation of the apparatus, but, in view of the great length of time devoted to it, the variety of methods of observation employed, the careful consideration of all sources of error, and the painstaking means adopted to eliminate them as far as possible from the measurements; it must, I think, be admitted as worthy of ranking with the work of the latter in point of accuracy, which is perhaps the highest praise that can be bestowed upon it. At the same time it must be remarked that in the opinion of the reviewer Boys' apparatus, modified so as to allow the suspended system to swing in a vacuum, and in some other minor particulars (see review already referred to), is capable of giving, under favorable circumstances, a much higher degree of accuracy than has been attained or can be attained with any other form as yet described or suggested. Had conditions of observation been favorable, Professor Boys would no doubt have succeeded in attaining the degree of accuracy which he himself has considered possible with it, *i. e.*, one part in ten thousand.

Dr. Braun's memoir is divided into six parts, with a supplement. Part I, the introduction, deals with the relation between the gravitation constant G, the mass of the earth M, and the mean density of the Earth D. Part II contains a full description of the apparatus and of the methods of measurement employed to determine its constants.

The apparatus used was that of Cavendish, considerably reduced in size, but still much larger than that employed by Boys. The attracted (swinging) system consisted of two gilded brass balls hung at the ends of a light balance-arm shaped frame of copper wire, which carried at the center a mirror about $3.^{cm}3$ in diameter, and which was suspended from a heavy tripod by a brass wire 104^{cm} long and $0.^{cm}0055$ in diameter. The upper end of the wire was attached to a torsion head, connected by clockwork with a shaft carrying a small magnet, which could be revolved by means of another magnet held outside the case, and the torsion head thus turned round without touching any part of the apparatus. This was a very necessary provision, as it was found that the creep of the index mark, due to the gradual change in the suspending wire, amounted during the course of the experiments, 1890–1895, to more than nine times the length of the scale. The whole suspended system was placed under a tall glass receiver connected with an air pump, by means of which it could be exhausted down to a pressure of about 2 to 5^{mm} of mercury. To determine the position of the suspended system a mirror inclined at 45° to the horizontal was placed in front of the mirror on the torsion rod, so as to reflect the light from the latter down through the base plate on to the horizontal objective of the observing telescope, of 46^{cm} focal length, placed just beneath. For convenience of observation a 45° prism was placed in the tube of the telescope, just behind the objective, so that the eyepiece of the latter was horizontal. Deflections were read by means of a fine glass scale $S_{,,}$, fixed in the focal plane of the eyepiece; the index mark being a line on a second brightly illuminated glass plate $S_{,}$, set in the lower side of the telescope tube, the light from which is thrown down the axis of the latter and on to the mirror on the torsion arm by a slip of optical glass placed just in front of $S_{,,}$. The distance of the scales from the mirror was so adjusted that a movement through one scale division corresponded to a movement of the torsion arm through an angle of 0.001 radian, *i. e.*, $3'.47$; as determined both by calculation and by observation with a theodolite. In order to observe a larger angular deflection than corresponded to simply one length of the scale, three index marks were placed on $S_{,}$ at a carefully determined distance apart. By observing the marks at opposite ends successively, a movement of the torsion arm corresponding to nearly ninety scale divisions, or about 5°, could be observed. The suspended system was set swinging by means of a light magnetized steel fork, the arms of which could

be brought against the beam on either side by means of an external magnet.

The moment of inertia of the torsion arm was determined both by calculation and by experiment in the usual manner, with very satisfactory agreement. The attracting system consisted of two spheres suspended outside the receiver from a graduated metal ring mounted so as to revolve concentrically with the axis of the torsion wire. Two sets of attracting masses were used, one set a pair of solid brass spheres, the other a pair of hollow iron spheres filled with mercury. The distance between the center of these masses was measured by means of an optical compass quite similar to that used by Professor Boys for a similar purpose, and the vertical positions were determined by means of a cathetometer. The principal dimensions of the apparatus were as follows:

Weights of small attracted masses (brass spheres),

$$M_1 = 54^{gm}.554$$
$$M_2 = 53\ .977$$
$$\text{mean} = 54\ .266$$

Distance between centers of small masses,

$$\text{right arm} = 12^{cm}.370$$
$$\text{left arm} = 12\ .243$$
$$24\ .613 \text{ at } 17^{\circ} \text{ C.}$$

Weights of large attracting masses (iron globes filled with mercury),

$$9184^{gm}.75$$
$$9107\ .57$$
$$\text{mean} = 9146\ .16$$

Distance between centers of large masses,

$$\text{right arm} = 20^{cm}.938$$
$$\text{left arm} = 20\ .800$$
$$41\ .738$$

The whole apparatus was mounted on a heavy stone slab in the corner of a room set apart for the work. It was protected from temperature changes, electrical effects, etc., by successive screens of tin and cloth.

Part III deals with the method of observation. Two independent methods were used, the first the original deflection method used by Cavendish (the same as used by Boys); the second the oscillation method first used by Reich. In the first method, which is too well known to need description, Dr. Braun determined the zero points, not by observations of successive elongations, as has usually been done, but by observing on a chronograph the times of transit in both directions of several divisions near the center of swing, and then determining from the successive differences the point about which the time of oscillation was the same in both directions. This was considered somewhat more accurate than the method of elongations. In the second method the attracting weights are placed in line with the centers of the attracted masses, in which position they act to increase the restoring force, and thus diminish the time of swing as compared with that observed when the masses are removed or turned at right angles to the first position. In the case of Dr. Braun's apparatus the time of vibration was changed from about 1251 seconds (for masses in line) to about 1296.6 seconds (for masses at right angles).

Part IV deals with the various corrections which have to be applied to the various observations, particularly those of the deflection and time of oscillation of the suspended system, for the various effects of elastic fatigue (*nachwerkung*), and "creep" of the torsion wire, damping of the residual air, changes in temperature, eccentricity of the suspended and attracting systems, etc. It would take too much space in this review to consider these various corrections in detail; suffice it to say that all possible sources of error seem to have been carefully considered and corrected for.

Part V gives in detail the various observations for the years 1892 and 1894, which were those finally chosen as the ones upon which to base the final determination of G and D. The observations of the previous years, 1887–1892, were rejected mainly on account of the fact that they had been made before the air pressure in the receiver had been sufficiently reduced to avoid irregularities of swing, etc. The observations in 1892 were made with a mean pressure of about 16^{mm}, and those of 1894 with a mean pressure of about 4^{mm} in the receiver.

The mean results for D by the deflection method for 1892 and 1894 were:

For 1892, $D = 5.531 \pm .003$, based on eleven complete observations.

For 1894, $D = 5.529 \pm .0016$, based on nine complete observations.

The mean results by the oscillation method were:

For 1892, $D = 5.523 \pm .0026$, based on fifteen complete observations.

For 1894, $D = 5.534 \pm .0032$, based on eleven complete observations.

Part VI and the supplement deal with the final discussion and correction of all the results obtained by both methods for the years 1892 and 1894. The most probable values of D as finally determined were:

By the deflection method, 1892, 5.529; 1894, 5.526.

By the oscillation method, 1892, 5.532; 1894, 5.531.

And for the mean result, $D = 5.527 \pm .001$.

Which gives for G

$$G = 6.6579 \times 10^{-8}.$$

This result is practically the same as that obtained by Professor Boys, *i. e.*, $G = 6.6576$. It will perhaps be remembered that in the review of Boys' work the writer pointed out that it seemed to him, from a consideration of the individual determinations upon which this latter value was based, that it was somewhat too low. Whether this be so or not, it is well not to be too much influenced by the striking agreement between these two determinations. Each is admitted to be uncertain by at least one and perhaps two units in the fourth place, so that the agreement to even the fifth figure is more likely to be a striking coincidence than an indication of real accuracy attained. Results obtained by other methods, notably the one obtained by Poynting, (1880–1891) by the balance method, have differed quite widely from the above, and while they are undoubtedly less accurate than the latter, so far as accidental errors of observation are concerned, it may be that the Cavendish method is subject to some constant source of error as yet unsuspected and undiscovered.

F. L. O. W.

Recent Publications.

A list of the titles of recent publications on astrophysical and allied subjects will be printed in each number of the Astrophysical Journal. In order that these bibliographies may be as complete as possible, authors are requested to send copies of their papers to both Editors. For convenience of reference, the titles are classified in thirteen sections.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

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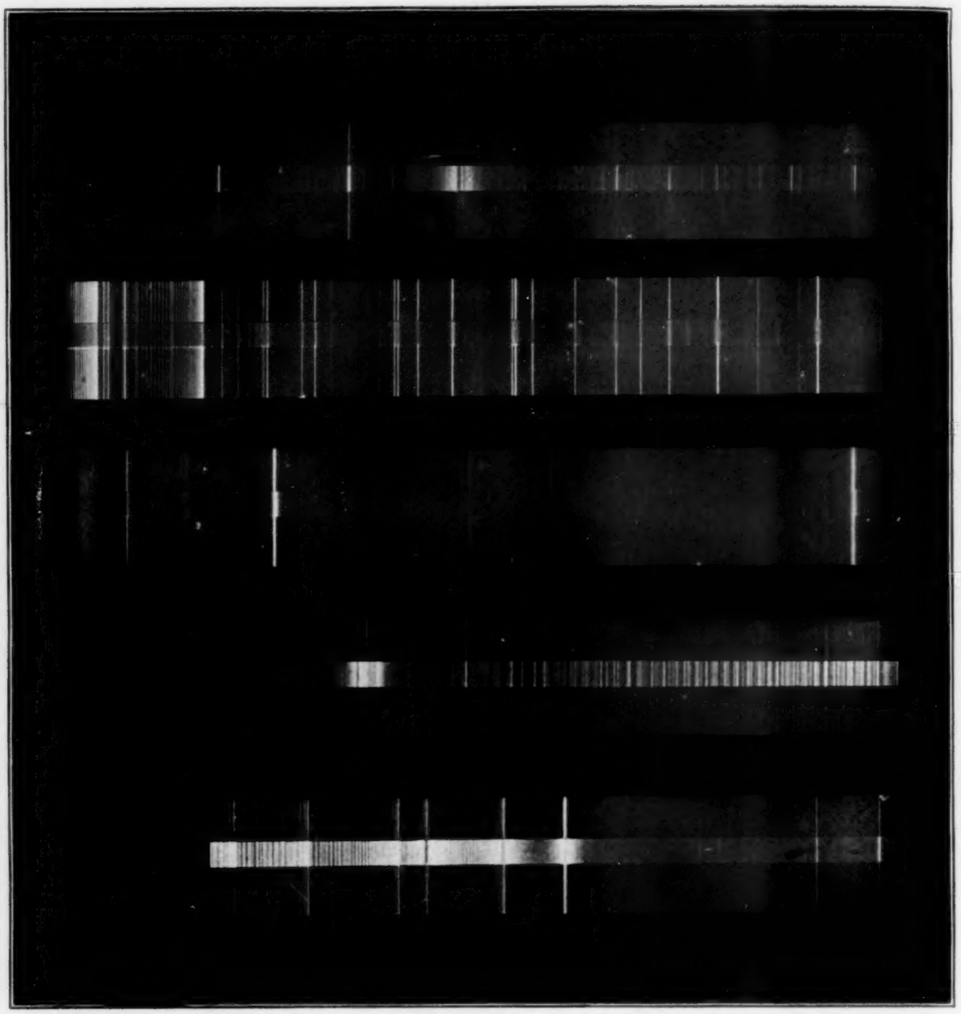

SHIFTS OF SPECTRAL LINES DUE TO PRESSURE

I. A pair of sodium lines which widen toward the violet, but shift toward the red.
II. New Concord Meteorite. All the iron lines are not equally shifted. Cyanogen bands unaffected.
III. Two classes of copper lines having different shifts.
IV. Large shifts of two potassium lines.
V. Shifts of the rubidium lines in the cyanogen band.